UNIT- I

Structure

1.0 Objectives

At the end of this unit, you will be able to:

- Appreciate the different aspects of mathematics;
- Be able to understand how mathematics has evolved from the time period;
- acquire a clear perspective of the nature and scope of mathematics education;
- explain the social and practical influences which have affected the growth of mathematics;
- acquire an understanding of some elementary mathematical concepts;
- see the relevance of mathematics as an essential part of the school curriculum both at elementary and secondary level;
- enumerate the far-reaching changes that have taken place in mathematics and in the teaching of mathematics; and
- get an insight into the various problems of teaching mathematics at the school stage

1.1 Introduction

Mathematics reveals hidden patterns that help us understand the world around us. Now much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behavior, and of social systems.

As a practical matter, mathematics is a science of pattern and order. Its domain is not molecules or cells, but numbers, chance, form, algorithms, and change. As a science of abstract objects, mathematics relies on logic rather than on observation as its standard of truth, yet employs observation, simulation, and even experimentation as means of discovering truth. The special role of mathematics in education is a consequence of its universal applicability. The results of mathematics--theorems and theories--are both significant and useful; the best results are also elegant and deep. Through its theorems, mathematics offers science both a foundation of truth and a standard of certainty.

In addition to theorems and theories, mathematics offers distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Experience with mathematical modes of thought builds mathematical power--a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives. Mathematics empowers us to understand better the information-laden world in which we live.

During the first half of the twentieth century, mathematical growth was stimulated primarily by the power of abstraction and deduction, climaxing more than two centuries of effort to extract full benefit from the mathematical principles of physical science formulated by Isaac Newton. Now, as the century closes, the historic alliances of mathematics with science are expanding rapidly; the highly developed legacy of classical mathematical theory is being put to broad and often stunning use in a vast mathematical landscape. Several particular events triggered periods of explosive growth. The Second World War forced development of many new and powerful methods of applied mathematics. Postwar government investment in mathematics, fueled by Sputnik, accelerated growth in both education and research. Then the development of electronic computing moved mathematics toward an algorithmic perspective even as it provided mathematicians with a powerful tool for exploring patterns and testing conjectures.

At the end of the nineteenth century, the axiomatization of mathematics on a foundation of logic and sets made possible grand theories of algebra, analysis, and topology whose synthesis dominated mathematics research and teaching for the first two thirds of the twentieth century. These traditional areas have now been supplemented by major developments in other mathematical sciences--in number theory, logic, statistics, operations research, probability, computation, geometry, and combinatorics.

In each of these subdisciplines, applications parallel theory. Even the most esoteric and abstract parts of mathematics--number theory and logic, for example--are now used routinely in applications (for example, in computer science and cryptography). Fifty years ago, the leading British mathematician G.H. Hardy could boast that number theory was the most pure and least useful part of mathematics. Today, Hardy's mathematics is studied as an essential prerequisite to many applications, including control of automated systems, data transmission from remote satellites, protection of financial records, and efficient algorithms for computation. In 1960, at a time when theoretical physics was the central jewel in the crown of applied mathematics, Eugene Wigner wrote about the ``unreasonable effectiveness'' of mathematics in the natural sciences: ``The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.'' Theoretical physics has continued to adopt (and occasionally invent) increasingly abstract mathematical models as the foundation for current theories. For example, Lie groups and gauge theories--exotic expressions of symmetry--are fundamental tools in the physicist's search for a unified theory of force.

During this same period, however, striking applications of mathematics have emerged across the entire landscape of natural, behavioral, and social sciences. All advances in design, control, and efficiency of modern airliners depend on sophisticated mathematical models that simulate performance before prototypes are built. From medical technology (CAT scanners) to economic planning (input/output models of economic behavior), from genetics (decoding of DNA) to geology (locating oil reserves), mathematics has made an indelible imprint on every part of modern science, even as science itself has stimulated the growth of many branches of mathematics.

Applications of one part of mathematics to another--of geometry to analysis, of probability to number theory--provide renewed evidence of the fundamental unity of mathematics. Despite frequent connections among problems in science and mathematics, the constant discovery of new alliances retains a surprising degree of unpredictability and serendipity. Whether planned or unplanned, the cross-fertilization between science and mathematics in problems, theories, and concepts has rarely been greater than it is now, in this last quarter of the twentieth century.

1.2 Nature and scope of Mathematics Education

Mathematics, like everything else that man has created, exists to fulfil certain human needs and desires. It is very difficult to say at what point of time in the history of mankind, and in which part of the world, mathematics had its birth. The fact that it has been steadily pursued for so many centuries, that it has attracted ever increasing attention and that it is now the dominant intellectual interest of mankind shows that it appeals very powerfully, to mankind. This conclusion is borne out by everything that we know about the origin of mathematics. More than 2,000 years before the beginning of the Christian era, both the Babylonians and the Egyptians were in possession of systematic methods of measuring space and time. They had the knowledge of rudimentary geometry and rudimentary astronomy. This rudimentary mathematics was formulated to meet the practical needs of an agricultural population. Their geometry resulted from the measurements made necessary by problems of land surveying Units of measurement, originally a stone or a vessel of water for weight, eventually became uniform over considerable areas under names which are now almost forgotten. Undoubtedly, similar efforts occurred in early times in the southern part of Central Asia along the Indus and Ganges rivers and in Eastern Asia Projects related to engineering, financing, irrigation, flood control, and navigation required mathematics. Again a usable calendar had to be developed to serve agricultural needs. *Zero* was defined and this at once led to positional notations for whole numbers and later to the same notation for fractions**.** The place value system which eventually developed was a gift of this period. These achievements and many more of a similar nature are the triumph of the human spirit. They responded to the needs of human society as it became more complex. Primitive men can hardly be said to have invented or discovered their arithmetic; they actually lived it. The men who shaped the stones in erecting the Temple of Mathematics were widely scattered, a few in Egypt, a few in India, and yet others in Babylon and China. These workmen confronted nature and worked in harmony with it. Their products, therefore though scattered in time and space, partook of the unity of nature.

Mathematics is something that the man has himself created to meet the cultural demands of time. Nearly every primitive tribe invented words to represent numbers. But it was only when ancient civilizations such as the Summerian, Babylonian, the Chinese and the Mayan developed trade, architecture, taxation and other civilized contracts that the number systems were developed. Thus, mathematics has grown into one of the most important cultural components of our society.

Our modern way of life would hardly have been possible without mathematics. Imagine trying to get through the day without using a number in some manner or the other. If a person lacks the ability to compute, he is as good as crippled. For instance, we need to know the time and tell the same. Telling the time is difficult and yet nearly everyone learns it. Soon, we shall lose an important experience of looking at the old fashioned clock with rotating hands, as we shall all be using digital readings to read time. A degree of estimation, not only in money but in 'weights and measures, is very important. Many of our daily routine chores involve sorting, ordering and organizing processes. We handle many mechanized devices which require geometrical or spatial skills. For travel, reading of maps, diagrams, interpreting scales becomes an essential part of our intellectual equipment. A knowledge of mathematics is useful to understand and interpret matters such as income tax and read information presented to us by the mass media in numerical form or in the form of graphs and understand the use of phrases such as rising prices, index, per capita income, inflation, stock market index etc. in ordinary day to day language. It is not necessary to provide an exhaustive list to prove the case in favour of "mathematics for survival" or "useful mathematics".

1.2.1 Historical Perspectives: Expanding the nature and scope of mathematics

Discussions of the nature of mathematics date back to the fourth century BC. Among the first major contributors to the dialogue were Plato and his student, Aristotle. Plato took the position that the objects of mathematics had an existence of their own, beyond the mind, in the external world. In doing so, Plato drew clear distinctions between the ideas of the mind and their representations perceived in the world by the senses. This caused Plato to draw distinctions between arithmetic-the theory of numbers-and logistics-the techniques of computation required by businessmen. In the *Republic* (1952a), Plato argued that the study of arithmetic has a positive effect on individuals, compelling them to reason about abstract numbers. Plato consistently held to this view, showing indignation at technicians' use of physical arguments to "prove" results in applied settings.

For Plato, mathematics came to "be identical with philosophy for modern thinkers, though they say that it should be studied for the sake of other things" (Aristotle, 1952, p. 510). This elevated position for mathematics as an abstract mental activity on externally existing objects that have only representations in the sensual world is also seen in Plato's discussion of the five regular solid~ in *Timaeus* (1952b) and his support and encouragement of the mathematical development of Athens (Boyer, 1968).

AristQtle, the student, viewed mathematics as one of three genera into which knowledge could be divided: the physical, the mathematical, and the theological: [Mathematics is the one] which shows up quality with respect to forms and local motions, seeking figure, number, and magnitude, and also place, time, and similar things. . . . Such an essence falls, as it were, between the other two, not only because it can be conceived both through the senses and without the senses. (Ptolemy, 1952, p. 5) This affirmation of the role of the senses as a source for abstracting ideas concerning mathematics was different from the view held by his teacher, Plato. Aristotle's view of mathematics was not based on a theory of an external, independent, unobservable body of knowledge. Rather it was based on experienced reality, where knowledge is obtained from experimentation, observation, and abstraction. This view supports the conception that one constructs the relations inherent in a given mathematical situation. In Aristotle's view, the construction of a mathematical idea comes through idealizations performed by the mathematician as a result of experience with objects. Thus, statements in applied mathematics are approximations of theorems in pure mathematics (K6rner, 1960). Aristotle attempted to understand mathematical relationships through the collection and classification of empirical results derived from experiments and observations and then by deduction of a system to explain the inherent relationships in the data. Thus, the works and ideas of Plato and Aristotle molded two of the major contrasting themes concerning the nature of mathematics. By the Middle Ages, Aristotle's work became known for its contributions to logic and its use in substantiating scientific claims. Although this was not contrary to the way in which Aristotle had employed his methods of logical reasoning, those who employed his principles often used them to argue against the derivation of evidence from empirical investigations. Aristotle drew clear lines between the ideal *forms* envisioned by Plato and their empirical realizations in worldly objects. The distinctions between these two schools of mathematical thought were further commented upon by Francis Bacon in the early 1500s when he separated mathematics into pure and mixed mathematics:

To the pure mathematics are those sciences belonging which handle quantity determinate, merely severed from any axioms of natural philosophy. . . , For many parts of nature can neither be invented with sufficient subtlety, nor demonstrated with sufficient perspicuity, nor accommodated unto use with sufficient dexterity, without the aid and intervening of the mathematics. (1952, p. 46) Similar discussions concerning the nature of mathematics were also echoed by Jean D'Alembert and other members of the French salon circle (Brown, 1988). Descartes worked to move mathematics back to the path of deduction from accepted axioms. Though experimenting himself in biological matters, Descartes rejected input from experimentation and the senses in matters mathematical because it might possibly delude the perceiver. Descartes's consideration of mathematics worked to separate it from the senses: For since the name "Mathematics" means exactly the same as "scientific study," , , . we see that almost anyone who has had the slightest schooling, can easily distinguish what relates to Mathematics in any question from that which belongs to the other sciences. . . . I saw consequently that there must be some general science to explain that element as a whole which gives rise to problems about order and measurement restricted as these are to no special subject matter. This, I perceived, was called "Universal Mathematics," not a far fetched designation, but one of long standing which has passed into current use, because in this science is contained everything on account of which the others are called parts of Mathematics. (1952, p. 7) This struggle between the rationalists and the experimentalists affected all branches of science throughout the 17th and 18^{th} centuries.

The German philosopher Immanuel Kant brought the discussion of the nature of mathematics, most notably the nature of geometry, back in to central focus with his *Critique of Pure Reason* (1952). Whereas he affirmed that all axioms and theorems of mathematics were truths, he held the view that the nature of perceptual space was Euclidean and that the contents of Euclidean geometry were a priori understandings of the human mind. This was in direct opposition to the emerging understandings of non-Euclidean geometry. The establishment of the consistency of non-Euclidean geometry in the mid-1800s finally freed mathematics from the restrictive yoke of a single set of axioms thought to be the only model for the external world. The existence of consistent non-Euclidean geometries showed the power of man's mind to construct new mathematical structures, free from the bounds of an externally existing, controlling world (Eves, 1981; Kline, 1972, 1985; Korner, 1960). This discovery, exciting as it was, brought with it a new notion of "truth," one buried in the acceptance of an axiom or a set of axioms defining a model for an area of investigation. Mathematicians immediately began to apply this new freedom and axiomatic method to the study of mathematics.

1.2.2 Late 19th and Early 20th Century Views

New investigations in mathematics, freed from reliance on experimentation and perception, soon encountered new problems with the appearance of paradoxes in the real number system and the theory of sets. At this point, three new views of mathematics arose to deal with the perceived problems. The first was the school of logicism, founded by the German mathematician Gottlob Frege in 1884. This school, an outgrowth of the Platonic school, set out to show that ideas of mathematics could be viewed as a subset of the ideas of logic. The proponents of logicism set out to show that mathematical propositions could be expressed as completely general propositions whose truth followed from their form rather than from their interpretation in a specific contextual setting. A N. Whitehead and Bertrand Russell (191(} 13) set out to show this in their landmark work, *Principia Mathematica.* This attempt was equivalent to trying to establish classical mathematics from the terms of the axioms of the set theory developed by Zermelo and FrankeL This approach, as that of Frege, was built on the acceptance of an externally existing mathematics, and hence was a direct outgrowth of the Platonic schooL Whitehead and Russell's approach failed through its inability to establish the axioms of infinity and choice in a state of complete generality devoid of context. This Platonic approach also failed because of the paradoxes in the system.

The followers of the Dutch mathematician L. E. J. Brouwer, on the other hand, did not accept the existence of any idea of classical mathematics unless it could be constructed via a combination of clear inductive steps from first principles. The members of Brouwer's school of thought, called the intuitionists, were greatly concerned with the appearance of paradoxes in set theory and their possible ramifications for all of classical mathematics. Unlike the logicists, who accepted the contents of classical mathematics, the intuitionists accepted only the mathematics that could be developed from the natural numbers forward through the mental activities of constructive proofs. This approach did not allow the use of the law of the excluded middle. This logical form asserts that the statement *PV -P* is true and makes proof by contradiction possible. In many ways, the ideas put forth by Brouwer were based on a foundation not unlike that professed by Kant. Brouwer did not argue for the "inspection of external objects, but [for] 'close introspection'" (K6rner, 1960, p. 120). This conception portrayed mathematics as the objects resulting from "valid" demonstrations. Mathematical ideas existed only insofar as they were constructible by the human mind. The insistence on construction placed the mathematics of the intuitionists within the Aristotelian tradition. This view took logic to be a subset of mathematics. The intuitionists' labors resulted in a set of theorems and conceptions different from those of classical mathematics. Under their criteria for existence and validity, it is possible to show that every real-valued function defined for all real numbers is continuous. Needless to say, this and other differences from classical mathematics have not attracted a large number of converts to intuitionism.

The third conception of mathematics to emerge near the beginning of the 20th century was that of formalism. This school was molded by the German mathematician David Hilbert. Hilbert's views, like those of Brouwer, were more in line with the Aristotelian tradition than with Platonism. Hilbert did not accept the Kantian notion that the structure of arithmetic and geometry existed as descriptions of a priori knowledge to the same degree that Brouwer did. However, he did see mathematics as arising from intuition based on objects that could at least be considered as having concrete representations in the mind. Formalism was grounded in the attempts to characterize mathematical ideas in terms of formal axiomatic systems. This attempt to free mathematics from contradictions was built around the construction of a set of axioms for a branch of mathematics that allowed for the topic to be discussed in a first-order language. Considerable progress was made in several areas under the aegis of formalism before its demise as a result of Kurt G6del's 1931 landmark paper. G6del (1931) established that it is impossible in axiomatic systems of the type Hilbert proposed to prove formally that the system is free of contradictions. G6del also demonstrated that it is impossible to establish the consistency of a system employing the usual logic and number theory if one uses only the major concepts and methods from traditional number theory. These findings ended the attempt to so formalize all of mathematics, though the formalist school has continued to have a strong impact on the development of mathematics (Benacerraf & Putnam, 1964; van Heijenoort, 1967; Snapper, 1979a, 1979b). The three major schools of thought created in the early 1900s to deal with the paradoxes discovered in the late 19^{th} century advanced the discussion of the nature of mathematics, yet none of them provided a widely adopted foundation for the nature of mathematics. All three of them tended to view the contents of mathematics as products. In logicism, the contents were the elements of the body of classical mathematics, its definitions, its postulates, and its theorems. In intuitionism, the contents were the theorems that had been constructed from first principles via "valid" patterns of reasoning. In formalism, mathematics was made up of the formal axiomatic structures developed to rid classical mathematics of its shortcomings. The influence of the Platonic and Aristotelian notions still ran as a strong undercurrent through these theories. The origin of the "product" -either as a pre-existing external object or as an object created through experience from sense perceptions or experimentation - remained an issue.

1.2.3 Modern Views and Mathematics

The use of a product orientation to characterize the nature of mathematics is not a settled issue among mathematicians. They tend to carry strong Platonic views about the existence of mathematical concepts outside the human mind. When pushed to make clear their conceptions of mathematics, most retreat to a formalist, or Aristotelian, position of mathematics as a game played with symbol systems according to a fixed set of socially accepted rules (Davis &Hersh, 1980). In reality, however, most professional mathematicians think little about the nature of their subject as they work within it. The formalist tradition retains a strong influence on the development of mathematics (Benacerraf &Putnam, 1964; Tymoczko, 1986). Hersh (1986) argues that the search for the foundations of mathematics is misguided. He suggests that the focus be shifted to the study of the contemporary practice of mathematics, with the notion that current practice is inherently fallible and, at the same time, a very public activity (Tymoczko, 1986). To do this, Hersh begins by describing the plight of the working mathematician. During the creation of new mathematics, the mathematician works as if the discipline describes an externally existing objective reality: But when discussing the nature of mathematics, the mathematician often rejects this notion and describes it as a meaningless game played with symbols. This lack of a commonly accepted view of the nature of mathematics among mathematicians has serious ramifications for the practice of mathematics education, as well as for mathematics itself.

The conception of mathematics held by the teacher has a strong impact on the way in which mathematics is approached in the classroom (Cooney, 1985). A teacher who has a formalist philosophy will present content in a structural format, calling on set theoretic language and conceptions (Hersh, 1986). Such a formalistic approach may be a good retreat for the individual who does not understand the material well enough to provide an insightful constructive view. Yet, if such formalism is *not* the notion carried by mathematicians, why should it dominate the presentation of mathematics in the classroom? To confront this issue, a discussion of the nature of mathematics must come to the foreground in mathematics education. Tymoczko and Hersh argue that what is needed is a new philosophy of mathematics, one that will serve as a basis for the working mathematician and the working mathematics educator.

According to Hersh, the working mathematician is not controlled by constant attention to validating every step with an accepted formal argument. Rather, the mathematician proceeds, guided by intuition, in exploring concepts and their interactions. Such a path places the focus on understanding as a guide, not long, formal derivations of carefully quantified results in a formal language.

This shift calls for a major change. Mathematics must be accepted as a human activity, an activity not strictly governed by anyone school of thought (logicist, formalist, or constructivist). Such an approach would answer the question of what mathematics is by saying that: Mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects). What are the main properties of mathematical activity or mathematical knowledge, as known to all of us from daily experience?

1. Mathematical objects are invented or created by humans.
2. They are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life.
3. Once created, mathematical objects have properties which are well determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them.(Hersh, 1986, p. 22)

The development and acceptance of a philosophy of mathematics carries with it challenges for mathematics and mathematics education. A philosophy should call for experiences that help mathematician, teacher, and student to experience the invention of mathematics. It should call for experiences that allow for the mathematization, or modeling, of ideas and events. Developing a new philosophy of mathematics requires discussion and communication of alternative views of mathematics to determine a valid and workable characterization of the discipline.

1.2.4 Pure and Applied Mathematics

The study of the history of mathematics does not answer the question "what is mathematics"? However, it provides a valuable perspective to understand the nature of mathematics. Mathematics has a cumulative growth from prehistoric times. This growth has been of two kinds : extrinsic in the form of primary discoveries and intrinsic development of the subject. The primary discoveries have been those of essential basic ideas, most of them gained by trial and error. Primarily, they were responses to human needs consistent with the body of knowledge already, existing before the emergence of the new ideas. They are true accretions. Secondly, in addition to accretions motivated by human needs, the cumulative development of mathematics has been due to its inner growth.

As Nunn has so well said, "Mathematical truths always have two sides or aspects. With the one they face and have contact with the world of outer realities lying in time and space, with the other they face and have contact with each other. Thus, the fact that equiangular triangles have proportional sides enables me to determine by draiing or by calculation the height of an Unscaleable mountain peak twenty miles away. This is the first or the outer aspect of that mathematical truth. On the other hand, I can deduce the truth itself with complete certainty from the assumed properties of congruent triangles. This is its second or inner aspect."

In brief, historically mathematics has grown largely as a result of

i) social needs, as shown in everyday life, commerce, science and technology,

ii) the intellectual need to connect together existing mathematics into a single logical framework or proof structure.

Thus, the word mathematics can be used in two distinct and different senses, i.e.,

i) the truths that are discovered, and

ii) the methods used to discover truths.

This distinction leads us to explore the question of pure and applied mathematics. In a classroom much of the mathematics we teach is applied mathematics in the sense that it relates directly to life's activities connected with buying, selling, trade, business, consumer applications, weighing, measuring etc. These applications of mathematics to the world around us can be extended to more technical ones. Mathematics has helped in analysing motion and in doing so, Newton created the calculus which became known as applied mathematics. More recently mathematical growth has been in areas such as operational research, linear programming, system analysis, statistics, all involving processes to handle numerical information in an increasingly technologically advanced world. The mathematical ideas we teach in schools develop over many years of study and become associated in our minds with all the applications and illustrations presented to explain them. It is always easier to explain what we can do with a concept in mathematics than to say what it is. A teacher has to answer questions such as "what is the use of this to us this?', or "why do we have to learn this?'If he/she fails to do so there will be many children who will not be able to see the point.

In pure mathematics we start from certain rules of inference, by which we call infer that if one proposition is true, then some other proposition is also true. These rules or inference constitute the major part of the principles of formal logic. For instance, we all know the axiom that in real numbers if $a > b$ and $b > c$, then $a > c$. Thus, from given propositions we conclude that some other proposition is true. We then take any hypothesis that seems amusing, and deduce its consequences. If our hypothesis is about anything, and not about some (one or more) particular person or thing, then our deductions constitute mathematics. Thus, mathematics may be defined as "the subject in which we never know what we are talking about, nor whether what we are saying is me" - *Bertrund Russell*

These ideas point out the abstract nature of mathematics. Mathematics deals with the application of arbitrary rules in an arbitrary situation which may or may not have significance in the world outside. It is a network of logical relationships. In school mathematics Euclidean geometry is essentially pure mathematics. A set of axioms and postulates are given and from them a body of definitions, theorems and propositions are derived. All pure mathematics is built up by combinations of primitive ideas of logic; its propositions are deduced from the general axioms of logic, such as the syllogism and the other rules of inference.

As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. The abstractions can be anything from strings of numbers to geometric figures to sets of equations. In addressing, say, "Does the interval between prime numbers form a pattern?" as a theoretical question, mathematicians are interested only in finding a pattern or proving that there is none, but not in what use such knowledge might have. In deriving, for instance, an expression for the change in the surface area of any regular solid as its volume approaches zero, mathematicians have no interest in any correspondence between geometric solids and physical objects in the real world.

A central line of investigation in theoretical mathematics is identifying in each field of study a small set of basic ideas and rules from which all other interesting ideas and rules in that field can be logically deduced. Mathematicians, like other scientists, are particularly pleased when previously unrelated parts of mathematics are found to be derivable from one another, or from some more general theory. Part of the sense of beauty that many people have perceived in mathematics lies not in finding the greatest elaborateness or complexity but on the contrary, in finding the greatest economy and simplicity of representation and proof. As mathematics has progressed, more and more relationships have been found between parts of it that have been developed separately—for example, between the symbolic representations of algebra and the spatial representations of geometry. These cross-connections enable insights to be developed into the various parts; together, they strengthen belief in the correctness and underlying unity of the whole structure.

Mathematics is also an applied science. Many mathematicians focus their attention on solving problems that originate in the world of experience. They too search for patterns and relationships, and in the process they use techniques that are similar to those used in doing purely theoretical mathematics. The difference is largely one of intent. In contrast to theoretical mathematicians, applied mathematicians, in the examples given above, might study the interval pattern of prime numbers to develop a new system for coding numerical information, rather than as an abstract problem. Or they might tackle the area/volume problem as a step in producing a model for the study of crystal behavior.

The results of theoretical and applied mathematics often influence each other. The discoveries of theoretical mathematicians frequently turn out—sometimes decades later—to have unanticipated practical value. Studies on the mathematical properties of random events, for example, led to knowledge that later made it possible to improve the design of experiments in the social and natural sciences. Conversely, in trying to solve the problem of billing long-distance telephone users fairly, mathematicians made fundamental discoveries about the mathematics of complex networks. Theoretical mathematics, unlike the other sciences, is not constrained by the real world, but in the long run it contributes to a better understanding of that world.

There is a very thin line dividing pure and applied concepts. On the one hand concepts of pure mathematics are formulated because of the need to apply them and on the other, every discovery or formulation has some application somewhere.

1.3 Axiomatic framework of Mathematics

The idea of an axiomatic structure in mathematics concerns the very foundation of mathematical reasoning. The origin of axiomatics can be traced to Euclid's Elements. Euclid introduced rigour by deducing his geometrical theorems from clearly stated assumptions embodied in his axioms and postulates. The essential components of an axiom system are ;

1. **Undefined terms** : These are derived in some way from experience and depend on intuition or imagination.

2. Axioms or postulates : These are assertions derived from undefined terms. They are formulated in terms of relationships for which no proofs are expected.

3. Propositions : Propositions derived from the axioms by logical reasoning.

Euclid's execution of this idea was defective as he inadvertently omitted to state all the assumptions which he subsequently utilized in his demonstrations

Peano's axiom system for deriving the natural numbers is another example of axiomatics.

Undefined terms : **“**0, "number", "successor"

Axioms/Postulates : A 1 **0** is a number.

A2 The successor of a number is anumber.

A3 No two numbers have the same successor.

A4 **0** is not the successor of any number.

A5 If P is a property such that (i) **0** has the property P, and

(ii) whenever n has the property P, the successor of n has the property P, then every number has the property P.

The 5th axiom is called "the principle of mathematical induction".

Mathematics is creative and intuition is the first step towards creativity. The analytical approach is needed to validate the new discoveries of intuition in a rigorous manner. According to the dictionary, intuition consists of the immediate apprehension, without the intervention of any reasoning process or knowledge or mental perception. Intuition is a mental act, a guess which gives a formulation or conclusion without going through a step by analysis.

An intuitive thinker arrives at an answer with very little awareness of how he she has reached it. In learning mathematics, the ability to visually dissect a pattern or a structure and guess a tentative generalization should be encouraged. When pupils are presented with the finished product which has been already formalised then their intuition suffers and they lose the opportunity to use an important tool in problem solving. No doubt, the intuition of pupils may not always be correct, nevertheless the teacher should have an open mind to the mistakes pupils make. This is because intuition is the essence of any non-rigorous method of solving problems.

In geometry, intuition helps to discover the proof of a result and the nature of mathematical proof is the next important question to consider. All propositions which mathematicians enunciate can be deduced one from the other by the rules of formal logic. Mathematics helps in developing logical reasoning, inductive and deductive thinking, analysis as well as evaluation skills. It also helps in developing skills of problem posing, visualisation, abstraction and problem solving. This is one of the many methods of proving results in geometry.

We now discuss some types of proofs in geometry.

Direct proof : It proceeds from propositions already accepted (axioms or theorems) via a chain of syllogism to the desired conclusion. Example (-a) (-b) = ab.

Indirect proof : (Or reductio ad absurdum) If p is to be proved, assume **not p** is true and hence derive a contradiction. Then, the assumption **not p** has to be false. So not **not p,** i.e., p is true. Example : $\sqrt{2}$ is irrational is proved in this way.

Proof using contrapositive : It p is of the form "a implies b", then prove **not** b implies **not a.**

Example : If two lines are cut by a transversal so that a pair of interior alternate angles are equal, the lines are parallel. The contrapositive is "If two lines cut by a transversal are not parallel, then the interior alternate angles are not equal".

Disproof by counter example : Used when the proposition conjectured is of universal form Simply exhibit a counter example.

Example : The sum of any two odd numbers is odd. This proposition is false, since 3 + 5 = 8 which is even.

1.3.1 Nature of any mathematical Inquiry

Using mathematics to express ideas or to solve problems involves at least three phases: (1) representing some aspects of things abstractly, (2) manipulating the abstractions by rules of logic to find new relationships between them, and (3) seeing whether the new relationships say something useful about the original things.

Abstraction and Symbolic Representation

Mathematical thinking often begins with the process of abstraction—that is, noticing a similarity between two or more objects or events. Aspects that they have in common, whether concrete or hypothetical, can be represented by symbols such as numbers, letters, other marks, diagrams, geometrical constructions, or even words. Whole numbers are abstractions that represent the size of sets of things and events or the order of things within a set. The circle as a concept is an abstraction derived from human faces, flowers, wheels, or spreading ripples; the letter A may be an abstraction for the surface area of objects of any shape, for the acceleration of all moving objects, or for all objects having some specified property; the symbol + represents a process of addition, whether one is adding apples or oranges, hours, or miles per hour. And abstractions are made not only from concrete objects or processes; they can also be made from other abstractions, such as kinds of numbers (the even numbers, for instance).

Such abstraction enables mathematicians to concentrate on some features of things and relieves them of the need to keep other features continually in mind. As far as mathematics is concerned, it does not matter whether a triangle represents the surface area of a sail or the convergence of two lines of sight on a star; mathematicians can work with either concept in the same way. The resulting economy of effort is very useful—provided that in making an abstraction, care is taken not to ignore features that play a significant role in determining the outcome of the events being studied.

Manipulating Mathematical Statements

After abstractions have been made and symbolic representations of them have been selected, those symbols can be combined and recombined in various ways according to precisely defined rules. Sometimes that is done with a fixed goal in mind; at other times it is done in the context of experiment or play to see what happens. Sometimes an appropriate manipulation can be identified easily from the intuitive meaning of the constituent words and symbols; at other times a useful series of manipulations has to be worked out by trial and error.

Typically, strings of symbols are combined into statements that express ideas or propositions. For example, the symbol A for the area of any square may be used with the symbol s for the length of the square's side to form the proposition $A = s^2$. This equation specifies how the area is related to the side—and also implies that it depends on nothing else. The rules of ordinary algebra can then be used to discover that if the length of the sides of a square is doubled, the square's area becomes four times as great. More generally, this knowledge makes it possible to find out what happens to the area of a square no matter how the length of its sides is changed, and conversely, how any change in the area affects the sides.

Mathematical insights into abstract relationships have grown over thousands of years, and they are still being extended—and sometimes revised. Although they began in the concrete experience of counting and measuring, they have come through many layers of abstraction and now depend much more on internal logic than on mechanical demonstration. In a sense, then, the manipulation of abstractions is much like a game: Start with some basic rules, then make any moves that fit those rules—which includes inventing additional rules and finding new connections between old rules. The test for the validity of new ideas is whether they are consistent and whether they relate logically to the other rules.

Application

Mathematical processes can lead to a kind of model of a thing, from which insights can be gained about the thing itself. Any mathematical relationships arrived at by manipulating abstract statements may or may not convey something truthful about the thing being modeled. For example, if 2 cups of water are added to 3 cups of water and the abstract mathematical operation 2+3 = 5 is used to calculate the total, the correct answer is 5 cups of water. However, if 2 cups of sugar are added to 3 cups of hot tea and the same operation is used, 5 is an incorrect answer, for such an addition actually results in only slightly more than 4 cups of very sweet tea. The simple addition of volumes is appropriate to the first situation but not to the second—something that could have been predicted only by knowing something of the physical differences in the two situations. To be able to use and interpret mathematics well, therefore, it is necessary to be concerned with more than the mathematical validity of abstract operations and to also take into account how well they correspond to the properties of the things represented.

Sometimes common sense is enough to enable one to decide whether the results of the mathematics are appropriate. For example, to estimate the height 20 years from now of a girl who is 5' 5" tall and growing at the rate of an inch per year, common sense suggests rejecting the simple "rate times time" answer of 7' 1" as highly unlikely, and turning instead to some other mathematical model, such as curves that approach limiting values. Sometimes, however, it may be difficult to know just how appropriate mathematical results are—for example, when trying to predict stock-market prices or earthquakes.

Often a single round of mathematical reasoning does not produce satisfactory conclusions, and changes are tried in how the representation is made or in the operations themselves. Indeed, jumps are commonly made back and forth between steps, and there are no rules that determine how to proceed. The process typically proceeds in fits and starts, with many wrong turns and dead ends. This process continues until the results are good enough.

But what degree of accuracy is good enough? The answer depends on how the result will be used, on the consequences of error, and on the likely cost of modeling and computing a more accurate answer. For example, an error of 1 percent in calculating the amount of sugar in a cake recipe could be unimportant, whereas a similar degree of error in computing the trajectory for a space probe could be disastrous. The importance of the "good enough" question has led, however, to the development of mathematical processes for estimating how far off results might be and how much computation would be required to obtain the desired degree of accuracy

1.3.2 Language of Mathematics

In teaching mathematics, the teacher uses ordinary language to communicate mathematical concepts and to clarify thoughts. Language is a means of gradually internalising experience to the point where actions can proceed in imagination without recourse to their physical repetition. Teaching of mathematics in the class is not only concerned with the computational knowledge of the subject but is also concerned with the selection of the mathematical content and communication leading to its understanding and application. So while teaching mathematics one should use the teaching methods, strategies and pedagogic resources that are much more fruitful in gaining adequate responses from the students than we have ever had in the past. We know that the teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in teaching-learning of mathematics.

For learning mathematical concepts children are initially engaged in activities with concrete materials, then encouraged to make audible descriptions and instructions - the concrete aids being withdrawn gradually until, finally, the concepts are internalised in verbal form. Thus language becomes a means of storing experience and facilitating problem solving. Effective learning of mathematical concepts does not result from mastery over activities alone.

It depends on how far teachers are successful in developing language or other symbolic representations, building links with past experiences to formulate corresponding abstractions or laws. The transition from concrete to abstraction depends upon explanations written in mathematical terms. Today a physicist (for that matter, any scientist) cannot pursue his or her studies without extensive use of mathematical language. Even subjects like biology, psychology, etc., which, used to be descriptive, are increasingly using mathematical notions. Persons studying the form and structure of language have also applied mathematics to explore it. Roger Bacon said "Mathematics is the gate and key of the sciences. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot view the other sciences or the things of the world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy."

Mathematics, thus, may be seen as **a** tool or a means of communication. Interesting studies of language difficulties experienced by children in mathematics have been made. Some features of mathematical language need special mention:

1. Mathematical language distinguishes between things and names of things. Number and numeral, and fraction and fractional numbers are a few examples.
2. Some common spoken words are used as technical terms and sometimes even in different contexts. For example, "variable" is used both as a noun and as an adjective; "root" is used as a root of an equation and as in square root, cube root, etc.
3. There are a variety of ways of calling a thing. For example, addition can be referred to as "find the sum", "find the total", "find the value", "find the whole", "how many in all?" etc.
4. Abbreviations (or labelling) are used. This usually helps in sustained thinking but sometimes they may not be in standard form or may be used only to avoid some steps in an algorithm. For example using gm, for gram is wrong; using cm is also not correct.
5. Frequently, auxiliary figures and markings are taught when new topics or operations are introduced. For example: to write carrying figure in addition; -> or "or" in writing equations. 5 m **x** 4 m = 20 sqm. is not correct because the multiplier is simply a number. it cannot be concrete. The correct way is (5 **x** 4) sqm.
6. Mathematical solutions emphasize a specific arrangement of steps in the solution, i.e., an algorithm to develop accuracy of thought and precision In quantitative matters.
7. Like all other languages, the language of mathematics has its own grammar. It has its own nouns, verbs, adjectives, etc.

The main characteristics of mathematical language are simplicity, accuracy and precision in contrast to ordinary language which can be ambiguous, vague and emotive. Special care is needed in formulating definitions. A good definition should satisfy the following conditions:

1. A definition should be consistent, i.e. it should convey the same meaning of the terms in all possible situations of the system.
2. A definition should not only consist of undefined terms or other previously defined terms. but also the common articles and connectives.
3. A definition should be stated clearly and precisely without redundancy.

1.4 Place of Mathematics in school education both at elementary and secondary levels

Mathematics is an important component of school education. Its influence has been so fundamental and widespread that being numerate is becoming more important than being literate.

The following values justify its position.

1.4.1 Social Aspects

The routine activities of daily life demand a mastery of number facts and number processes. To read with understanding much of the materials in newspapers requires considerable mathematical vocabulary. A few such terms are percent, discount, commission, dividend, invoice, profit and loss, wholesale and retail, taxation, etc. As civilization is becoming more complex, many terms from the electronic media and computers are being added. Certain decisions require sufficient skill and understanding of quantitative relations. The ability to sense problems, to formulate them specifically and to solve them accurately requires systematic thinking.

To understand many institutions and their management problem, a quantitative viewpoint (modelling) is necessary. It is illuminating to hear from an economist, an architect, an engineer, an aviator, or a scientist what in mathematics is helpful to them as workers.' Many vocations need mathematical skills.

The child should gain an appreciation of the role played by mathematics in many fields of work. Since, scientific knowledge and technology are linked with the progress and prosperity of a nation; we should be able to appreciate the role of mathematics in acquiring these.

Mathematics has helped in bringing together the countries of the world which are separated from each other physically. Mathematics helped man to discover the mysteries of nature and to overcome superstitions and ignorance.

1.4.2 Mathematical Aspects

- Mathematics teaches us how to analyse a situation, how to come to a decision, to check ' thinking and its results, to perceive relationships, to concentrate, to be accurate and to be systematic in our work habits.
- Mathematics develops the ability to perform necessary computations with accuracy and reasonable speed. It also develops an understanding of the processes of measurement and of the skill needed in the use of instruments of precision.
- Mathematics develops the ability to

  a) make dependable estimates and approximations,

  b) devise and use formulae, rules of procedure and methods of making comparisons,

  c) represent designs and spatial relations by drawings, and

  d) arrange numerical data systematically and to interpret information in graphic or tabular form.

1.4.3 Applications of Mathematics

'The history of mathematics is the story of the progress of civilizations and culture.

"Mathematics is the mirror of civilization". A country's civilization and culture is reflected in the knowledge of mathematics it possesses. Mathematics helps in the preservation, promotion and transmission of cultures. Various cultural arts like poetry, painting, drawing, and sculpture utilise mathematical knowledge. Mathematics has aesthetic or pleasure value. Concepts like symmetry, order, similarity, form and size form the basis of all work of art and beauty. All poetry and music utilizes mathematics. Quizes, puzzles, and magic squares are both entertaining and challenging to thought. Hence, the teaching of mathematics is inevitable in our schools may be at any level.

1.4.4 Mathematics in elementary and secondary education

The curriculum is a tool to achieve the proposed objectives of teaching a subject. In broader terms it is the sum total of all the experiences of the pupil that he undergoes in school, home and in informal contacts between the teacher and the pupil. The importance and content of curriculum in mathematics both at secondary level can be listed as an outcome of the values and importance discussed above. These are briefly mentioned here.

1. A good mathematics programme should present suitable learning experiences to foster common needs as citizens and special needs as an individual. The main consideration should be given to desirable pupil growth within the overall purposes of elementary and secondary education. It should therefore :
   a) Try to develop basic concepts, competency and skills of mathematics
   b) The fundamentals about number sense, identifying shapes and spatial thinking, basic algebra, geometry, fractions and decimals and data handling capacity.
   c) emphasize those behaviours which will fit pupils better for useful service to their community and for ethical living;
   d) gain an appreciation of the importance and power of mathematics in the development of society;
   e) discover vocational possibilities
   f) develop the ability to analyse and to solve problems of everyday life situations; and
   g) develop proficiency in mathematics as a method of communication.

2. In addition to competence in arithmetic (computational skills in four fundamental processes), elementary ideas of algebra, proof and sets are being included.

3. Pure rote learning until facts are memorised almost mechanically is now giving way to discovering numbers as a property of the manyness of a set, ordering the numbers, mapping, building a system of numeration and using expanded notation for fostering conceptual understanding.

4. Greater attention is now being given to preparing pupils for the subsequent study of mathematics at higher levels.

5. Many curriculum developers use Piaget's stages of learning for planning learning situations.

These stages are

a) pre-conceptual stage from birth to age four or five years;

b) intuitive stage from ages five to eight years;

c) concrete operational stage from ages eight to thirteen;

d) 'formal operational stage from ages twelve or thirteen onwards, where learning of the reasoning process, appraising, forming hypothesis, generalizing, deducing and proving is emphasized.

The Gestalt approach to learning places emphasis on relation and restructuring of mathematical situation. It is exemplified by the so-called "multiple embodiment" procedure where many different aspects or approaches to a mathematical concept are presented at the same period of learning so that by gradual reinforcement and restructuring, there results a more general yet precise mental formulation of the concept.

6. The laboratory approach and activity oriented is more popular in many places. Here in the classroom pupils try to "discover mathematics". The classroom is equipped with all types of materials: electronic devices, measuring equipment, models, geoboards, blocks, cards, charts, etc..The children may work in groups or individually on projects. Experimentation and discovery are encouraged.

7. The school is but one phase of education for living, and as such should provide an orderly sequence that will enable the pupils to maintain steady growth. Knowledge and familiarity with the work of succeeding grades and with higher education is necessary. , Within the field of mathematics, the correlation between various parts should be maintained. Algebra and arithmetic can be correlated with geometry. Algebra and geometry can be correlated with trigonometry. The tendency to keep various parts in "watertight compartments" needs to be given up.

8. Attention must be paid also to the relationship of mathematics with other subjects such as physics, chemistry, biology, geography or social sciences.

9 a) Work must be child-centred, ‘not teacher-dominated, The pupils must participate and perform. The teacher should motivate, organize and direct the children to learn.

b) The activities should have significance to the learner and should be based on practical life situations.

c) Problem methods are more favourable teaching situations. ,

10. Content should be decided on the basis of mental age, interest level, present usefulness to learner and future use to learner.

11. Provision should be made for the below-average learners. Each child should get functional experiences on his own level of ability and interests so that he can succeed in developing mathematical competence of value in practical life situations.

1.5 Problems of teaching Mathematics at the school stage

Any analysis of mathematics education in our schools will identify a range of issues as problematic. We structure our understanding of these issues around the following four problems which we deem to be the core areas of concern:

1. A sense of fear and failure regarding mathematics among a majority of children,
2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time,
3. Crude methods of assessment that encourage perception of mathematics as mechanical computation, and
4. Lack of teacher preparation and support in the teaching of mathematics.

Each of these can and need to be expanded on, since they concern the curricular framework in essential ways.

1.5.1 Fear and Failure

If any subject area of study evokes wide emotional comment, it is mathematics. While no one educated in Tamil would profess (or at the least, not without a sense of shame) ignorance of any Tirukkural, it is quite the social norm for anyone to proudly declare that (s)he never could learn mathematics. While these may be adult attitudes, among children (who are compelled to pass mathematics examinations) there is often fear and anxiety. Mathematics anxiety and ‘math phobia’ are terms that are used in popular literature.

In the Indian context, there is a special dimension to such anxiety. With the universalisation of elementary education made a national priority, and elementary education a legal right, at this historic juncture, a serious attempt must be made to look into every aspect that alienates children in school and contributes towards their non-participation, eventually leading to their dropping out of the system. If any subject taught in school plays a significant role in alienating children and causing them to stop attending school, perhaps mathematics, which inspires so much dread, must take a big part of the blame. Such fear is closely linked to a sense of failure. By Class III or IV, many children start seeing themselves as unable to cope with the demands made by mathematics. In high school, among children who fail only in one or two subjects in year-end examinations and hence are detained, the maximum numbers fail in mathematics. This statistic pursues us right through to Class X, which is when the Indian state issues a certificate of education to a student. The largest numbers of Board Exam failures also happen in mathematics. There are many perceptive studies and analyses on what causes fear of mathematics in schools. Central among them is the cumulative nature of mathematics. If you struggle with decimals, then you will struggle with percentages; if you struggle with percentages, then you will struggle with algebra and other mathematics subjects as well. The other principal reason is said to be the predominance of symbolic language. When symbols are manipulated without understanding, after a point, boredom and bewilderment dominate for many children, and dissociation develops. Failure in mathematics could be read through social indicators as well. Structural problems in Indian education, reflecting structures of social discrimination, by way of class, caste and gender, contribute further to failure (and perceived failure) in mathematics education as well. Prevalent social attitudes which see girls as incapable of mathematics, or which, for centuries, have associated formal computational abilities with the upper castes, deepen such failure by way of creating self-fulfilling expectations. A special mention must be made of problems created by the language used in textbooks, especially at the elementary level. For a vast majority of Indian children, the language of mathematics learnt in school is far removed from their everyday speech, and especially forbidding. This becomes a major force of alienation in its own right.

1.5.2 Disappointing Curriculum

Any mathematics curriculum that emphasises procedure and knowledge of formulas over understanding is bound to enhance anxiety. The prevalent practice of school mathematics goes further: a silent majority give up early on, remaining content to fail in mathematics, or at best, to see it through, maintaining a minimal level of achievement. For these children, what the curriculum offers is a store of mathematical facts, borrowed temporarily while preparing for tests. On the other hand, it is widely acknowledged that more than in any other content discipline, mathematics is the subject that also sees great motivation and talent even at an early age in a small number of children. These are children who take to quantisation and algebra easily and carry on with great facility. What the curriculum offers for such children is also intense disappointment. By not offering conceptual depth, by not challenging them, the curriculum settles for minimal use of their motivation. Learning procedures may be easy for them, but their understanding and capacity for reasoning remain under exercised.

1.5.3 Crude Assessment

We talked of fear and failure. While what happens in class may alienate, it never evokes panic, as does the examination. Most of the problems cited above relate to the tyranny of procedure and memorization of formulas in school mathematics, and the central reason for the ascendancy of procedure is the nature of assessment and evaluation. Tests are designed (only) for assessing a student's knowledge of procedure and memory of formulas and facts, and given the criticality of examination performance in school life, concept learning is replaced by procedural memory. Those children who cannot do such replacement successfully experience panic, and suffer failure. While mathematics is the major ground for formal problem solving in school, it is also the only arena where children see little room for play in answering questions.

Every question in mathematics is seen to have one unique answer, and either you know it or you don't. In Language, Social Studies, or even in Science, you may try and demonstrate partial knowledge, but (as the students see it), there is no scope for doing so in mathematics. Obviously, such a perception is easily coupled to anxiety. Amazingly, while there has been a great deal of research in mathematics education and some of it has led to changes in pedagogy and curriculum, the area that has seen little change in our schools over a hundred years or more is evaluation procedures in mathematics. It is not accidental that even a quarterly examination in Class VII is not very different in style from a Board examination in Class X, and the same pattern dominates even the end-of chapter exercises given in textbooks. It is always application of some piece of information given in the text to solve a specific problem that tests use of formalism. Such antiquated and crude methods of assessment have to be thoroughly overhauled if any basic change is to be brought about.

1.5.4 Inadequate Teacher Preparation

More so than any other content discipline, mathematics education relies very heavily on the preparation that the teacher has, in her own understanding of mathematics, of the nature of mathematics, and in her bag of pedagogic techniques. Textbook-centred pedagogy dulls the teacher's own mathematics activity. At two ends of the spectrum, mathematics teaching poses special problems. At the primary level, most teachers assume that they know all the mathematics needed, and in the absence of any specific pedagogic training, simply try and uncritically reproduce the techniques they experienced in their school days. Often this ends up perpetuating problems across time and space.

At the secondary and higher secondary level, some teachers face a different situation. The syllabi have considerably changed since their school days, and in the absence of systematic and continuing education programmes for teachers, their fundamentals in many concept areas are not strong. This encourages reliance on 'notes' available in the market, offering little breadth or depth for the students.

While inadequate teacher preparation and support acts negatively on all of school mathematics, at the primary stage, its main consequence is this: mathematics pedagogy rarely resonates with the findings of children's psychology. At the upper primary stage, when the language of abstractions is formalised in algebra, inadequate teacher preparation reflects as inability to link formal mathematics with experiential learning. Later on, it reflects as incapacity to offer connections within mathematics or across subject areas to applications in the sciences, thus depriving students of important motivation and appreciation.

1.5.5 Other Systemic Problems

We wish to briefly mention a few other systemic sources of problems as well. One major problem is that of compartmentalisation: there is very little systematic communication between primary school and high school teachers of mathematics, and none at all between high school and college teachers of mathematics. Most school teachers have never even seen, let alone interacted with or consulted, research mathematicians. Those involved in teacher education are again typically outside the realm of college or research mathematics.

Another important problem is that of curricular acceleration: a generation ago, calculus was first encountered by a student in college. Another generation earlier, analytical geometry was considered college mathematics. But these are all part of school curriculum now. Such acceleration has naturally meant pruning of some topics: there is far less solid geometry or speherical geometry now. One reason for the narrowing is that calculus and differential equations are critically important in undergraduate sciences, technology and engineering, and hence it is felt that early introduction of these topics helps students proceeding further on these lines. Whatever the logic, the shape of mathematics education has become taller and more spindly, rather than broad and rounded.

While we have mentioned gender as a systemic issue, it is worth understanding the problem in some detail. Mathematics tends to be regarded as a 'masculine domain'. This perception is aided by the complete lack of references in textbooks to women mathematicians, the absence of social concerns in the designing of curricula which would enable children questioning received gender ideologies and the absence of reference to women's lives in problems. A study of mathematics textbooks found that in the problem sums, not a single reference was made to women's clothing, although several problems referred to the buying of cloth, etc. Classroom research also indicates a fairly systematic devaluation of girls as incapable of 'mastering' mathematics, even when they perform reasonably well at verbal as well as cognitive tasks in mathematics. It has been seen that teachers tend to address boys more than girls, which feeds into the construction of the normative mathematics learner as male. Also, when instructional decisions are in teachers' hands, their gendered constructions colour the mathematical learning strategies of girls and boys, with the latter using more invented strategies for problem-solving, which reflects greater conceptual understanding. Studies have shown that teachers tend to attribute boys' mathematical 'success' more to ability, and girls' success more to effort. Classroom discourses also give some indication of how the 'masculinising' of mathematics occurs, and the profound influence of gender ideologies in patterning notions of academic competence in school. With performance in mathematics signifying school 'success', girls are clearly at the losing end.

1.6 Let us Sum up

In this particular unit we have tried to broadly discuss the nature and scope of mathematics. The historical development of mathematics has tried to clarify not only on the scope and nature of mathematics but also that how innate nature of inquiry and need satisfaction has lead towards the development of mathematics. Mathematics has evolved as a simple tool of estimation and calculation to a building block of architectural science and then an indispensable area of human development. It has also been explained as an academic discipline while explaining its axiomatic nature as its building blocks. Further it has also been explained about language of mathematics and how mathematical inquiry starts from abstraction and systematically analyzed and solution are finalized. Again it has been discussed in detail about the place of mathematics in school both at elementary and secondary education and its significance. Lastly the unit completes with a discussion on the different problems in mathematics education under different subareas.

1.7 Check Your Progress

Now just try to go through the following questions and check your progress.

Q.1 Write the nature and scope of school mathematics?

Q.2 How mathematics has evolved from primitive days to the modern day scenario?

Q.3 Describe the axiomatic framework of mathematics?

Q.4 Mathematics is an abstract subject and helps in development of logical thinking and problem solving. Explain?

Q.5 Justify the place of school mathematics at elementary and secondary level?

Q.6 Mathematics education in school have very critical issues to be address. Discuss?

1.8 Suggested Readings

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UNIT-II

Structure

1.0 Objectives

At the end of this unit, you will be able to:

- Understand the aspects of aims and objectives of mathematics teaching;
- Differentiate between general objectives and stage specific objectives of teaching mathematics;
- List out the objectives of mathematics teaching at different stages of school learning;
- Acquire the understanding of Bloom's taxonomy of educational objectives;
- Get an insight of Blooms taxonomy of educational objectives at the elementary and secondary level;
- Write the instructional objectives of mathematics teaching at the elementary and secondary level;
- See the relevance of mathematics to the different values it poses;
- Enumerate different educational values of mathematics education.
- Give an insight into different ccurricular perspectives at different stages in mathematics.

1.1 Introduction

Education is essential for achieving certain ends and goals. Learning of various subjects at school level are different means to achieve these goals. Mathematics as always held a key position in the school curriculum, because it has been considered knowledge indispensable to every human being. However, the knowledge of mathematics merely not meant for computational arithmetic and geometrical measurements but also played an important role in the education of all people. With the changing scenario of the World, mathematics has occupied an important role even in non- mathematical areas such as social-sciences, medical sciences etc.. With this new role, aims and objectives of teaching mathematics at school level have under gone tremendous changes from time to time according to the needs of scientific and technological oriented society.

The term 'aims of teaching mathematics,' stands for the goal or broad purpose that needs to be fulfilled by the teaching of mathematics in the general scheme of education. Aims are like ideals and their attainment needs long term planning. Therefore, they are divided into some definite functional and workable units. The specific objectives are those short term, immediate goals and purposes that may be achieved within the specified classroom transactions. The instructional objectives are very pin-pointed objectives to be achieved during a teaching-learning setup.

This Unit, we will discuss different aims and objectives of mathematics teaching at school level. In addition to that we will also try to make you aware of different educational values related to school mathematics education.

1.2 Aims and objectives of teaching mathematics at school stage

Before we start the teaching of a subject, it is important for us to know as to why we are going to teach it. The process of teaching can be kept on right lines only with the help of clear cut aims. Aimlessness in teaching would result in the wastage of time, energy and other resources.

What would be the aims of teaching mathematics in our schools? The answer requires the knowledge of all the advantages that can be drawn from the teaching of mathematics. These aims will be based on the educational values of the subject. Aims and values are interrelated and interdependent. Aims help in te realization of the values possessed by a subject.

Education is imparted for achieving certain ends and goals. Various subjects of the school curriculum are different means to achieve these goals. The term aims of teaching mathematics stands for the goals, targets or broader purposes that may be fulfilled by the teaching of mathematics in the general scheme of education. Aims are like ideals. Their attainment needs a long-term planning. Their realization is not easy task. Therefore, they are divided into some definite, functional and workable units named as objectives. The objective are those short-term, immediate goals or purposes that may be achieved within the specified classroom situations. The aims are broken into specified objectives to provide definite learning experiences for bringing about desirable behavioural changes.

1.2.1 Objectives of teaching mathematics at entire school stage

Aims of teaching mathematics are to be framed in the light of the educational values of the subject. Value is the spring-board of aim. We know that mathematics has wide applications in our daily life. It has great cultural and disciplinary values. Thus we may mention the aims of teaching mathematics as under:

Aims

1. To enable the students to solve mathematical problems of daily life. We have to select the content and methods of teaching so that the students are able to make use of their learning of mathematics in daily life.

2. To enable the students to understand the contribution of mathematics to the development of culture and civilisation.

3. To develop thinking and reasoning power of the students.

4. To prepare a sound foundation needed for various vocations. Mathematics is needed in various professions such as those of engineers, bankers, scientists, accountants, statisticians etc.

5. To prepare the child for further learning in mathematics and the related fields. School mathematics should also aim at preparing him for higher learning in mathematics.

6. To develop in the child desirable habits and attitudes like habit of hard work, self-reliance, concentration and discovery.

7. To give the child an insight into the relationship of different topics and branches of the subject.

8. To enable the child to understand popular literature. He should be so prepared that he finds no handicap in understanding mathematical terms and concepts used in various journals, magazines, newspapers etc.

9. To teach the child the art of economic and creative living.

10. To develop in the child rational and scientific attitude towards life.

Objectives

Aims of teaching mathematics are genially scope whereas objectives of the subject are specific goals leading ultimately to the general aims of the subject. The objectives of teaching mathematics in school can be described as under:

A. Knowledge and Understanding Objectives

Through mathematics, a pupil acquires the knowledge of the following:

(i) He learns mathematical language, for example, mathematical symbols, formulae figures, diagrams, definitions etc.

(ii) He understands and uses mathematical concepts like concept o area, volume, number, direction etc.

(iii) He learns the fundamental mathematical ideas, processes, rules and relationships.

(iv) He understands the historical background of various topics and contribution of mathematicians.

(v) He understands the significance and use of the units of measurement]

B. Skill Objectives

Mathematics develops the following skills:

(i) The child learns to express thoughts clearly and accurately.

(ii) He learns to perform calculations orally.

(iii) He develops the ability to organise and interpret the given data

(iv) He learns to reach accurate conclusions by accurate and logic reasoning.

(v) He learns to analyse problems and discover fundamental relationships.

(vi) He develops speed and accuracy in solving problems.

(vii) He develops the skill to draw accurate geometrical figures,

(viii) He develops the ability to use mathematical apparatuses and tools skilfully.

C. Application Objectives

The students will be able to develop the following application objectives

(i) He is able to solve mathematical problems independently.
(ii) He makes use of mathematical concepts and processes in everyday life.
(iii) He develops ability to analyse, to draw inferences, and to generalise from the collected data and evidences.
(iv) He can think and express precisely, exactly, and systematically by making proper use of mathematical language.
(v) He develops the ability to use mathematical knowledge in the learning of other subjects especially sciences.
(vi) He develops the students' ability to apply mathematics in his future vocational life.

D. Appreciation and Interest Objectives

The child learns to appreciate:

(i) The contribution of mathematics to the development of various subjects and occupations.

(ii) The role played by mathematics in modern life.

(iii) The mathematical type of thought which serves as model for scientific thinking in other fields.

(iv) The rigour and power of mathematical processes and accrue of results.

(u) The cultural value of mathematics.

(vi) The value of mathematics as leisure time activity.

E. Attitude Objectives

Mathematics helps in the development of following attitudes:

(i) The child develops the attitude of systematically pursuing a task to completion.

(ii) He develops heuristic attitude. He tries to make independent discoveries. (Hi)' He develops the habit of logical reasoning.

(iv) He is brief and precise in expressing statements and results,

(v)He develops the habit of verification.

(vi) He develops power concentration and independent thinking. (vii) He develops habit of self-reliance.

We have discussed the aims and objectives of teaching mathematics in general. The teacher should carefully choose the objectives regarding a particular topic. The nature of students will also be kept in view. The aims of teaching and learning mathematics are to encourage and enable students to:

- recognize that mathematics permeates the world around us
- appreciate the usefulness, power and beauty of mathematics
- enjoy mathematics and develop patience and persistence when solving problems
- understand and be able to use the language, symbols and notation of mathematics
- develop mathematical curiosity and use inductive and deductive reasoning when solving problems
- develop a positive attitude towards learning Mathematics
- perform mathematical operations and manipulations with confidence, speed and accuracy
- think and reason precisely, logically and critically in any given situation
- develop investigative skills in Mathematics
- identify, concretise, symbolise and use mathematical relationships in everyday life

- comprehend, analyse, synthesise, evaluate, and make generalizations so as to solve mathematical problems
- Collect, organize, represent, analyse, interpret data and make conclusions and predictions from its results
- apply mathematical knowledge and skills to familiar and unfamiliar situations
- appreciate the role, value and use of Mathematics in society
- develop willingness to work collaboratively
- acquire knowledge and skills for further education and training
- communicate mathematical ideas
- become confident in using mathematics to analyse and solve problems both in school and in real-life situations
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others
- develop a critical appreciation of the use of information and communication technology in mathematics
- appreciate the international dimension of mathematics and its multicultural and historical perspectives.

1.2.2 Objectives of teaching mathematics at elementary stage

The objectives at the elementary stage can be supplemented as given:

A. Knowledge and understanding objectives

1. Develops the knowledge and understanding of mathematical concepts like number, units of measurement, size, shape, direction, distance, grouping sub-grouping and fractions.
2. Develops the knowledge and understanding of mathematical facts and processes like place value of numbers, meaning and significance of zero, four fundamental operations, percentage, unitary method, menstruation, etc.
3. Develops the knowledge and understanding of mathematical terms and symbols like digits, fractions, percentage, etc.
4. Develops the knowledge and understanding of fundamental mathematical relationships.

B. Skills objectives

The students develops the following skills :

1. Ability in counting, reading and writing of numbers.
2. Skill in four fundamental operations dealing with integral numbers and fractions.
3. A reasonable speed, accuracy and neatness in oral and written computational work.
4. Technique of solving problems which involve elementary mathematical processes and simple calculations.
5. Skill in the use of mathematical tables.
6. Proficiency in making quantitative estimate of size and distance.

C Application objectives

1. He is able to solve both oral and written mathematical problems independently.
2. He applies elementary mathematical concepts and processes in every day life.

D Attitude objectives

1. Develops self-confidence for solving elementary mathematical problems.
2. While solving mathematical problems, he tries to read it carefully, analyses it, collects all the known evidences and then draw proper inferences.
3. Develops the habits of neatness, regularity, honesty and truthfulness.
4. Develops the habits of logical thinking and objective reasoning.

E Appreciation and interest objectives

1. Develops interest in the learning of mathematics.
2. Appreciate the contribution of mathematicians and gets inspiration from their work.
3. Appreciates the power of computational skills.
4. Appreciates and takes interest in using his knowledge of mathematics in solving problems of daily life.
5. Appreciates the recreational value of mathematics and learns to utilize his leisure time properly.

1.2.3 Objectives of teaching mathematics at secondary stage

The objectives of teaching mathematics at secondary stage are as follows and are very similar to the objectives at the entire school stage

A. Knowledge and understanding objectives

1. He develops the knowledge and understanding of mathematical facts, concept and abstractions.
2. He develops the knowledge and understanding complex geometrical figures.

3. He develops the knowledge and understanding of polynomials, linear equations, and factorization.
4. He develops the knowledge and understanding of similarity, congruency and trigonometry

B. Skill objectives

1. He develops the skills of solving mathematical problems of secondary stage
2. He develops the skills of problem solving to deal with the word problems involving one or more variable.
3. He develops skills of inductive and deductive reasoning in solving geometrical problems.
4. Skills in finding out different trigonometric ratios values of fundamental angles.

C. Application objectives

1. He learns the application of mathematics in his day to day life problems.
2. He is able to apply mathematical ability to his social situation, vocational, occupational and recreational life.

D. Attitude objectives

1. He gains confidence and competence in the learning of mathematics.
2. Develops the habit of analytical thinking in day to day life situation.

E. Appreciation and interest objectives

1. He enjoys mathematical problem of every type.
2. He appreciates the use of mathematics in the entire daily life situation.
3. He justifies every one with the importance of mathematics in daily life.
4. He appreciates the utilitarian value of mathematics in life.

1.3 Blooms taxonomy of educational objectives interpreted for mathematics

Bloom's Taxonomy, a educational tool developed by Benjamin S. Bloom (1913-1999) that ranks the relative cognitive complexity of various educational objectives. This taxonomy is often used as an aid when create test questions and assignments and objectives of a subject. Following the description, you will find Lindsey Shorser's interpretation of Bloom's Taxonomy in the context of mathematical understanding with examples drawn from school level topics.

Bloom's Taxonomy of Cognitive Skills:

• Knowledge - retention of terminology, facts, conventions, methodologies, structures, principles, etc.
• Comprehension - grasping of meaning, translation, extrapolation, interpretation of facts, making comparisons, etc.
• Application - problem solving, usage of information in a new way
• Analysis - making inferences and supporting them with evidence, identification of patterns
• Synthesis - derivation of abstract relations, prediction, generalization, creation of new ideas
• Evaluation - judgement of validity, usage of a set of criteria to make conclusions, discrimination

Questions that encourage each of these skills often begin with:
• Knowledge: List, define, describe, show, name, what, when, etc.
• Comprehension: Summarize, compare and contrast, estimate, discuss, etc.
• Application: Apply, calculate, complete, show, solve, modify, etc.
• Analysis: Separate, arrange, classify, explain, etc.
• Synthesis: Integrate, modify, substitute, design,create, What if..., formulate, generalize, prepare, etc.
• Evaluation: Assess, rank, test, explain, discriminate, support, etc.

Taxonomy of Cognitive Objectives -1950s-developed by Benjamin Bloom 1990s-Lorin Anderson (former student of Bloom) revisited the taxonomy. The names of six major categories were changed from *noun* to *verb* forms. As the taxonomy reflects different forms of **thinking** and thinking is an *active* process, verbs were more accurate.
A mathematics teacher found that by using Bloom's higher levels --analyzing, evaluating and creating --when questioning students during Math, helps them become better problem solvers. Asking students to explain their Math answers, using words, drawings or diagrams and numbers (equations), is an excellent way to assess if they truly understand the concept taught. Having students extend and explain a number pattern engages them in higher level thinking skills, too. Students come up with their own questions about a lesson to help each other. This is difficult at first and they usually just ask knowledge questions. But with practice, students begin to ask better questions --questions to which they can explain the answers.
Questioning should be used purposefully to achieve well-defines goals. Typically a teacher would vary the level of questions within a single lesson.

Usually questions at the lower levels are appropriate for:	Questions at higher levels of the taxonomy are usually most appropriate for:
• evaluating students' preparation and comprehension. • diagnosing students' strengths and weaknesses. • reviewing and/or summarizing content	• encouraging students to think more deeply and critically. • problem solving, encouraging discussions. • stimulating students to seek information on their own.

*Bloom's **Revised** Taxonomy: The new terms are defined as:*

Remembering: Retrieving, recognizing, and recalling relevant knowledge from long-term memory. eg. find out, learn terms, facts, methods, procedures, concepts.

Understanding: Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Understand uses and implications of terms, facts, methods, procedures, concepts.

Applying: Carrying out or using a procedure through executing, or implementing. Make use of, apply practice theory, solve problems, use information in new situations.

Analyzing: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing. Take concepts apart, break them down, analyze structure, recognize assumptions and poor logic, evaluate relevancy.

Evaluating: Making judgments based on criteria and standards through checking and critiquing. Set standards, judge using standards, evidence, rubrics, accept or reject on basis of criteria.

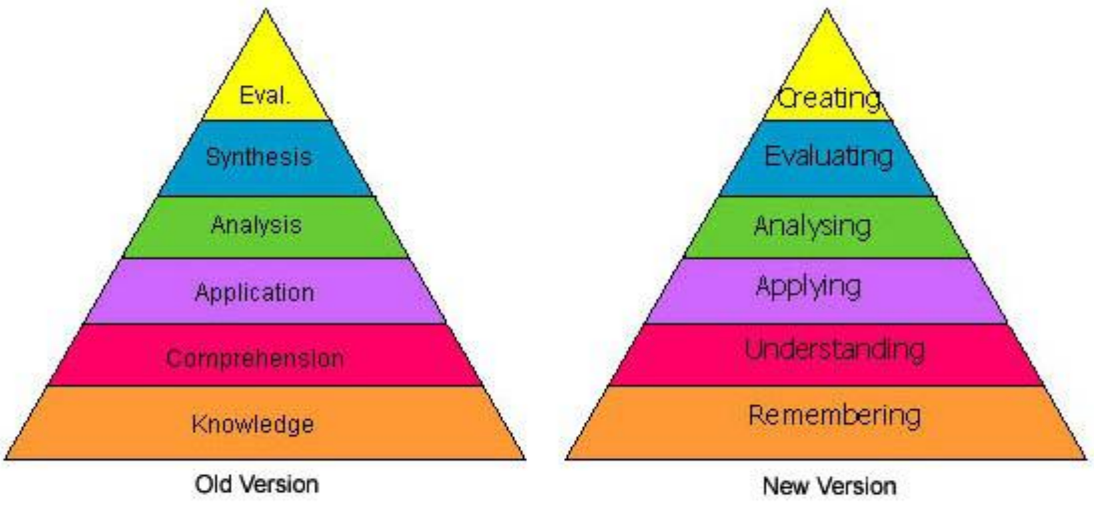
Creating: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing. Put things together; bring together various parts; write theme, present speech, plan experiment, put information together in a new & creative way.

Bloom's **Revised** Taxonomy: Mathematics

	New Terms	**Actions**	***Learning Activities***
Higher-order thinking	**Creating** (Putting together ideas or elements to develop an original idea or engage in creative thinking).	Designing Constructing Planning Producing Inventing Devising Making	***Creating:*** *(*Generating new ideas, products, or ways of viewing things) *How could we determine the number of pennies in a jar without counting them? Apply and integrate several different strategies to solve a mathematical problem. Design a new monetary system or an experiment for establishing ...* Designing, constructing, planning, producing, inventing. *Invent a machine to do a specific task. Develop a menu for a new healthy foods restaurant.*
	Evaluating (Judging the value of ideas, materials	Checking Hypothesizing Critiquing	***Evaluating:*** (Judging the value of a product for a given purpose, using definite criteria) *Develop a proof ... and justify each step ..., Using a definition ... determine ...* Justifying a decision or course

	and methods by developing and applying standards and criteria).	Experimenting Judging Testing Detecting Monitoring	of action, checking, hypothesizing, critiquing, experimenting, judging. *What criteria would you use to evaluate if your answer is correct? Prepare a list of criteria to judge… Evaluate expressions.*
	Analyzing (Breaking information into its component elements to explore relationships).	Comparing Organizing Deconstructin g Attributing Outlining Structuring Integrating	***Analyzing:*** *(*Breaking information into parts to explore understandings and relationships) *Given a math word problem, determine the strategies that would be necessary to solve it. Write a paragraph describing the relationship ..., How does ... compare to …*Comparing, organizing, deconstructing, interrogating, finding *Design a survey to find out ... Graph your results. Use a Venn Diagram to show how two topics are the same and different. Translate between visual representations, sentences, and symbolic notation. Make predictions based on experimental or statistical data.*
Lower-order thinking	**Applying** (Using strategies, concepts, principles and theories in new situations).	Implementing Carrying out Using Executing	***Applying:*** (Using information in concrete situations) *Compute the area of actual circles. Use the given graph to ..., Choose and describe the best method to ...* Using information in another familiar situation, implementing, carrying out, using, executing. *Draw a diagram which shows these fractions or take photographs of the fractions. Determine measures of central tendency and dispersion Write a journal entry. Write an explanation about this topic for others.*
	Understanding (Understanding of given information).	Interpreting Exemplifying Summarizing Inferring Paraphrasing Classifying Comparing Explaining	***Understanding:*** (Grasping the meaning of material) *Given the mathematical formula for the area of a circle, paraphrase it using your own words. Select the graph that illustrates* Explaining ideas or concepts Interpreting, summarizing, paraphrasing, classifying, explaining *Find items that you can use to show the fractions. Retell or write in your own words… Report to the class… Write a summary report of the event.*
	Remembering (Recall or recognition of specific information).	Recognizing Listing Describing Identifying Retrieving Naming Locating Finding	***Remembering:*** (Remembering previously learned material) *State the formula for the area of a circle. State the rule of ..., Explain and use the procedure for ...* Recalling information, recognizing, listing, describing, retrieving, naming, finding, locating *List the fractions you know and can show. List the attributes of your shape. Make a concept map of the topic. Make a chart showing…*

Bloom's **Revised** Taxonomy: Mathematics



Fractions with Bloom's Revised Taxonomy	3D shapes with Bloom's Revised Taxonomy
Remembering List the fractions you know and can show. ***Understanding*** Find items that you can use to show the fractions. ***Applying*** Draw a diagram which shows these fractions or take photographs of the fractions. ***Analysing*** Design a survey to find out which fractions are easy and which are hard. Graph your results. ***Evaluating*** Choose a diagram or picture to represent the hardest fractions to use in a game. ***Creating*** Create a power point presentation game for others to play.	***Remembering*** List the attributes of your shape. ***Understanding*** Find items that you can use to show the shape. ***Applying*** Draw a diagram or take a photograph of the shape. ***Analysing*** Identify where the shape is found in the classroom and school. ***Evaluating*** Tell why your shape is used in the places it is. ***Creating*** Create an item that includes all or part of your shape. Draw and label your design.

1.4 Instructional objectives at different levels

Though Bloom's Taxonomy of educational objectives in the cognitive domain as six levels of remembering, understanding , applying, analysing, evaluating and creating, as adapted to mathematics, it has only three, levels, namely, remarkably, understanding and applying . Here routine exercises and applications are included under knowledge and understanding depending on the complexity and applying level objectives, include non-routine application, analysis, creating and evaluation.

In view of the above, for writing instructional objectives we have employed following levels in the cognitive, affective and psychomotor domains: (1) Remembering, (2) Understanding, (3) Applying, (4) Skills, (5) Appreciation, (6) Interest, and (7) Attitudes.

1.4.1 Instructional objectives at elementary level

1. **Remembering Objectives:** To remember terms, facts, concepts, symbols, definitions, principles, and formulae of mathematics and also applications which are routine in nature requiring probably only substitution in a formula.

Specifications: To demonstrate the achievement of above objectives, the pupil:

1.1 Recall the terms, facts, definitions, formulae, concepts, processes, etc.

1.2 Recognize formulae, figure, concepts, procedures and processes, etc.

1.3 Solves routine types of problems using a formula

2. **Understanding Objectives:** To develop understanding of concepts, principles and processes of mathematics, to apply principles and processes in routine situations.

Specifications: To demonstrate the achievements of the above objectives, the pupil:

2.1 Gives illustrations and detect errors in statement, formulae or figure and correct them

2.2 Explain concepts, principles and configuration

2.3 Discriminate between closely related concepts and principles

2.4 Classifies as per given criteria

2.5 Identifies relationship among the given data

2.6 Translate variable statement in to symbolic relationships and vice-versa

2.7 Estimate the result

2.8 Interpret the given charts, graphs and data

2.9 Verify properties

2.10 Solves routine types of problem using concepts and processes

3. **Applying Objectives :** To apply the acquired knowledge and understanding of mathematics in unfamiliar situation or new situations.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

3.1 Analyses and solves non-routine types of problems

3.2 Finds out the adequacy and relevance of the given data

3.3 Establishes relationship among the given data

3.4 Selects the appropriate method for solving a problem

3.5 Suggests alternative methods of solutions

3.6 Gives justification to the method of solution

4. **Skill Objectives :** To acquire different skills (a) computing (b) drawing geometrical figure and graphs (c) reading tables graphs, charts, etc.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

4.1 Carries out oral calculations with ease and speed

4.2 Does written calculations with ease and speed

4.3 Handles geometrical instruments with ease and proficiency

4.4 Measures accurately

4.5 Draws free hand diagrams with ease

4.6 Draws figure accurately

4.7 Draws figure according to scale

5. **Appreciation Objectives :** To appreciate the use of mathematics in day-to day life and in other disciplines.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

5.1 Appreciates the use of mathematics in other disciplines

5.2 Appreciates the symmetry of figures and patterns

5.3 Appreciates qualities like brevity and exactness through the study of mathematics

6. Interest Objectives: To develop interest in mathematics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

6.1 Solve mathematical puzzles

6.2 Participates in thed activitirs of mathematical club

6.3 Reads additional material in mathematics

6.4 Formulates additional problems.

7. Attitude Objectives: To develop scientific attitude triough the study of mathamatics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

7.1 Examines all the aspects of a problem

7.2 Point out errors boldly if convinced

7.3 Accepts errors without hesitation

7.4 Respects the opinion of others.

1.4.2 Instructional objectives at secondary level

1. Remembering Objectives : To acquire the knowledge of terms, facts, concepts, symbols, definitions, principles, and formulae of mathematics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

1.1 Recall the terms, facts, definitions, formulae, concepts, processes

1.2 Recognise formulae, figure, concepts, procedures and processes

2. Understanding Objectives : To develop understanding of concepts, principles and processes of mathematics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

2.1 Gives illustration

2.2 detects errors in statement, formulae or figure and correct them

2.3 Explains concepts, principles and configuration

2.4 Discriminate between closely related concepts and principles

2.5 Classifies as per given criteria

2.6 Identifies relationship among the given data

2.7 Translate variable statement in to symbolic relationships and vice-versa

2.8 Estimate the result

2.9 Reads and interpret the given chart, graphs and data

2.10 Verify properties, indicates hypothesis

2.11 Solves routine types of problem using concepts, principles, etc.

2.12 Reads and interprets data from a given table

2.13 Uses calculator and computer

3. Applying Objectives : To apply the acquired knowledge and understanding of mathematics in unfamiliar situation or new situations.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

3.1 Analyses and solves non-routine types of problems

3.2 Finds out the adequacy and relevance of the given data

3.3 Establishes relationship among the given data

3.4 Selects the appropriate method for solving a problem

3.5 Suggests alternative methods of solutions

3.6 Gives justification to the method of solution

4. Skill Objectives : To acquire different skills (a) computing (b) drawing geometrical figure and graphs (c) reading tables graphs, charts, etc.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

4.1 Does written calculations with ease and speed

4.2 Handles geometrical instruments with ease and proficiency

4.3 Measures accurately

4.4 Draws free hand diagrams with ease

4.5 Draws figure accurately and according to the scale

4.6 Reads data/ table with speed and accuracy

5. Appreciation Objectives : To appreciate the use of mathematics in day-to day life and in other disciplines.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

5.1 Appreciates the use of mathematics in other curricular areas

5.2 Appreciates the symmetry of figures and patterns

5.3 Appreciates qualities like brevity and exactness

5.4 Appreciates the contributions of mathematicians in general and Indian mathematicians in particular.

6. Interest Objectives: To develop interest in mathematics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

6.1 Solve mathematical puzzles

6.2 Participates in the activities of mathematical club

6.3 Reads additional material in mathematics

6.4 Formulates additional problems.

7. Attitude Objectives: To develop scientific attitude through the study of mathematics.

Specifications : To demonstrate the achievements of the above objectives, the pupil:

7.1 Examines all the aspects of a problem

7.2 Point out errors boldly if convinced

7.3 Accepts errors without hesistation

7.4 Respects the opinion of others.

Example 1

Topic: Perimeter

Objectives

- To reinforce the idea of perimeter as boundary of the figure among learners using different activities.
- To enable learners to calculate perimeter of closed figures (using Geoboard).
- To enable the learner to use the formuala to calculate/ solve problems related to perimeter

Example 2

Topic : Circumference of a circle

Objectives

- To enable the students to understand the concept of circle.
- To enable learners to find the relationship between the circumference and the diameter of a circle.
- To enable learners to solve problems related to circumference of the circle using its formula
- To enable learners to differentiate between area and perimeter of a circle.

Example 3

Topic : Algebraic expressions

Objectives

- To enable learners to identify a variable in an algebraic expression.
- To enable learners to state the meaning of term ‘variable’ and ‘constant’
- To enable learners to define an algebraic expression.
- To enable learners to state the components of an algebraic expression.

- To enable learners to translate a given statement into an algebraic expression
- To enable learners to identify terms of an algebraic expression.
- To enable learners to differentiate between like and unlike terms.
- To enable learners to state the meaning of coefficient of a term and hence, enable them to write coefficients of the terms in an algebraic expression.
- To enable learners to classify algebraic expressions into monomials, binomials and trinomials, etc.

1.5 Values of mathematics education

Knowledge of educational values elps the teacher to avoid aimlessness in teaching. Value is the springboard of aim and vice-versa. There is nothing controversial about the two. One aims at a thing, because one values it : or by aiming at a thing, one shall taste it value. " We aim at teaching mathematics in the light of its aims, we shall realize its values." Aimless teaching will realize no values.

Broadly speaking there are three main considerations for which a child is sent to school. Education must contribute to the acquirement of these values :

i) Knowledge and skill
ii) Intellectual habits and power
iii) Desirable attitude and ideals

These three values can be called utilitarian, disciplinary and cultural values of education respectively. However in addition to that we will discuss here social and recreational values also.

1.5.1 Utilitarian Values

One cannot do without the use of fundamental processes of this subject in daily life. A common man can get on sometimes very well without learning how to read and write, but he can never pull on without learning how to count and calculate. Any person ignorant of mathematics will be at the mercy of others and will be easily cheated. The knowledge of its fundamental processes and the skill to use them are the preliminary requirements of a human being these days.

A person may belong to the lowest or the highest class of the society, but he utilizes knowledge of mathematics in one form or another. Not to speak of an engineer, businessman, an industrialist, a banker, a financier, a finance minister, a planner or a boss og any concern, even a labourer has got to calculate his wages, make purchases from the market, and adjust te expenditure to his income. If he is more sensible, he will believe in saving also for the rainy day. Whosoever earns and spends uses mathematics; and there cannot be anybody who lives without earning and spending.

In many occupations such as accountancy, banking, shop-keeping, business, tailoring, carpentry, taxation, insurance, computer applications, postal jobs, by which the basic needs of man are fulfilled, indirect or direct use of mathematics is made.

Counting, notation, addition, subtraction, multiplication, division, weighing, measuring, selling, buying and many more are simple and fundamental processes of mathematics which have got an immense practical value or utilitarian values. The knowledge and skill in these processes can be provided in an effective and systematic manner only by teaching mathematics in schools.

1.5.2 Cultural Values

The understanding of the world in which we live, of the civilization to which we belong and of the culture of which we are very profound, requires the understanding of scientific ands social principles the development of which he depends, in turn, upon mathematics principles. It has been truly said that "Mathematics is the mirror of civilization." Mathematics has got its cultural value, and this value is steadily increasing day by day. It helped man to overcome difficulties in the way of his progress. It has helped a major role in the bringing him to such advanced stage of development. The prosperity of man and his cultural advancement have depended considerably upon the advancement of mathematics.

Modern civilization owes its advancement to the progress of different occupations such as agriculture, engineering, surveying, medicine, industry, navigation, railroad building, etc. These occupations build up culture and they are its backbone. But one solid not forget that mathematics contributes and has contributed extensively to the advancement of theses occupations. Therefore mathematics shapes culture as a playback pioneer. Perhaps, the modern materialistic attitude in everything is the outcome of the deep influence of mathematics on life and culture.

Some of the important aspects of cultural heritage have been preserved in the form of mathematical knowledge only and learning of mathematics is the only medium to pass on this heritage to the coming generations. It is one of the repositories of precious and valued heritage.

Mathematics is also a pivot for cultural arts, such as music, sculpture, poetry, dance and painting. It might not be altogether a matter of chance that the Greeks, the greatest geometers were very successful in fine arts.

1.5.3 Social Values

Mathematics plays an important role in the organization and maintenance of our social structure. Society is the result of the inter-relations of individuals. It consists of big and small groups within each group. Mathematics enables us to understand the inter-relations of individuals and the possibilities of various groups.

Society is a phenomena of balancing and counter balancing of various social forces. Mathematics helps in creating a social order in this phenomena. It regulates the functioning of the society in many ways. Social conditions like justice, fairplay, competition, symmetry, harmony, etc. have often to be described in mathematical terms for the purpose of clarity.

Mathematics helps in the formation of social norms and their implementation. The dominance of materialistic outlook in our society is one of the chief attributes of mathematics. Our monetary dealings are a major domain of our social dealings and relations. We earn a social status by virtue of our economic status and behavior. The social status is governed by our property, income, bank balance and economic potential. Social security bound to imply economic security.

The ideas like manpower planning have originated partly due to the influence of mathematics. The statistical data and the census provide bases for short-term and long range planning for the welfare of the society.

1.5.4 Recreational Values

Mathematics as a subject has also lots of scope of recreational activities. Mathematics leads in developing of different interest and habits which further leads for recreational activities. Lots of recreational activities through the different subjects, arts and aesthetics, painting, poetry and literature mathematics creates scope for them. It also help in planning different leisure time games, healthy practices, managing a function, planning a trip, field study, etc.

The recreational values of mathematics can also be seen through the application of mathematics in different co-curricular activities of the schools like time-table management, academic plan, annual day planning etc.

1.5.5 Disciplinary Values

Mathematics has also to be taught for its disciplinary and intellectual value. It has to aim at providing training to the mind of the learner and developing intellectual habits in them. Mathematical knowledge is exact, real and pure. It trains and disciplines the mind. It develops powers of reasoning and thinking, and reduces reliance on rote memory. Learning in mathematics possesses certain features which help the learner in developing characteristics of discipline like accuracy, simplicity, certainty of results, originality, reasoning, self-evaluation and other by the products of these characteristics like concentration, truthfulness, seriousness, etc.

1.6 Curricular perspectives at different stages in mathematics

Acknowledging the existence of choices in curriculum is an important step in the institutionalization of education. Hence, when we speak of shifting the focus from content to learning environments, we are offering criteria by which a curriculum designer may resolve choices. For instance, visualization and geometric reasoning are important processes to be ensured, and this has implications for teaching algebra. Students who ‘blindly’ manipulate equations without being able to visualize and understand the underlying geometric picture cannot be said to have understood. If this means greater coverage for geometric reasoning (in terms of lessons, pages in textbook), it has to be ensured. Again, if such expansion can only be achieved by reducing other (largely computational) content, such content reduction is implied.

Below, while discussing stage-wise content, we offer many such inclusion /exclusion criteria for the curriculum designer, emphasizing again that the recommendation is not to dilute content, but to give importance to a variety of processes. Moreover, we suggest a principle of postponement: in general, if a theme can be offered with better motivation and applications at a later stage, wait for introducing it at that stage, rather than go for technical preparation without due motivation. Such considerations are critical at the secondary and higher secondary stages where a conscious choice between breadth and depth is called for. Here, a quotation from William Thurston is appropriate: *The long-range objectives of mathematics* *education would be better served if the tall shape of mathematics were de-emphasized, by moving away from a standard sequence to a more diversified curriculum with more topics that start closer to the ground. There have been some trends in this direction, such as courses in finite mathematics and in probability, but there is room for much more.*

1.6.1 Curricular perspectives at elementary stage in mathematics

Any curriculum for primary mathematics must incorporate the progression from the concrete to the abstract and subsequently a need to appreciate the importance of abstraction in mathematics. In the lowest classes, especially, it is important that activities with concrete objects form the first step in the classroom to enable the child to understand the connections between the logical functioning of their everyday lives to that of mathematical thinking. Mathematical games, puzzles and stories involving number are useful to enable children to make these connections and to build upon their everyday understandings. Games – not to be confused with open-ended play - provide nondidactic feedback to the child, with a minimum amount of teacher intervention. They promote processes of anticipation, planning and strategy.

While addressing number and number operations, due place must be given to non-number areas of mathematics. These include shapes, spatial understanding, patterns, measurement and data handling. It is not enough to deal with shapes and their properties as a prelude to geometry in the higher classes. It is important also to build up a vocabulary of relational words which extend the child's understanding of space. The identification of patterns is central to mathematics. Starting with simple patterns of repeating shapes, the child can move on to more complex patterns involving shapes as well as numbers. This lays the base for a mode of thinking that can be called algebraic. A primary curriculum that is rich in such activities can arguably make the transition to algebra easier in the middle grades. Data handling, which forms the base for statistics in the higher classes, is another neglected area of school mathematics and can be introduced right from Class I.

Children come equipped with a set of intuitive and cultural ideas about number and simple operations at the point of entry into school. These should be used to make linkages and connections to number understanding rather than treating the child as a tabula rasa. To learn to think in mathematical ways children need to be logical and to understand logical rules, but they also need to learn conventions needed for the mastery of mathematical techniques such as the use of a base ten system. Activities as basic as counting and understanding numeration systems involve logical understandings for which children need time and practice if they are to attain mastery and then to be able to use them as tools for thinking and for mathematical problem solving. Working with limited quantities and smaller numbers prevents overloading the child's cognitive capacity which can be better used for mastering the logical skills at these early stages. Operations on natural numbers usually form a major part of primary mathematics syllabi. However, the standard algorithms of addition, subtraction, multiplication and division of whole numbers in the curriculum have tended to occupy a dominant role in these. This tends to happen at the expense of development of number sense and skills of estimation and approximation. The result frequently is that students, when faced with word problems, ask "Should I add or subtract? Should I multiply or divide?" This lack of a conceptual base continues to haunt the child in later classes. All this strongly suggests that operations should be introduced contextually. This should be followed by the development of language and symbolic notation, with the standard algorithms coming at the end rather than the beginning of the treatment.

Fractions and decimals constitute another major problem area. There is some evidence that the introduction of operations on fractions coincides with the beginnings of fear of mathematics. The content in these areas needs careful reconsideration. Everyday contexts in which fractions appear, and in which arithmetical operations need to be done on them, have largely disappeared with the introduction of metric units and decimal currency. At present, the child is presented with a number of contrived situations in which operations have to be performed on fractions. Moreover, these operations have to be done using a set of rules which appear arbitrary (often even to the teacher), and have to be memorized - this at a time when the child is still grappling with the rules for operating on whole numbers. While the importance of fractions in the conceptual structure of mathematics is undeniable, the above considerations seem to suggest that less emphasis on operations with fractions at the primary level is called for.

Mathematics is amazingly compressible: one may struggle a lot, work out something, perhaps by trying many methods. But once it is understood, and seen as a whole, it can be filed away, and used as just a step when needed. The insight that goes into this compression is one of the great joys of mathematics. A major goal of the upper primary stage is to introduce the student to this particular pleasure.

The compressed form lends itself to application and use in a variety of contexts. Thus, mathematics at this stage can address many problems from everyday life, and offer tools for addressing them. Indeed, the transition from arithmetic to algebra, at once both challenging and rewarding, is best seen in this light.

A consolidation of basic concepts and skills learnt at primary school is necessary from several points of view. For one thing, ensuring numeracy in all children is an important aspect of universalization of elementary education. Secondly, moving from number sense to number patterns, seeing relationships between numbers, and looking for patterns in the relationships bring useful life skills to children. Ideas of prime numbers, odd and even numbers, tests of divisibility etc. offer scope for such exploration. Algebraic notation, introduced at this stage, is best seen as a compact language, a means of succinct expression. Use of variables, setting up and solving linear equations, identities and factoring are means by which students gain fluency in using the new language.

The use of arithmetic and algebra in solving real problems of importance to daily life can be emphasized. However, engaging children’s interest and offering a sense of success in solving such problems is essential.

A variety of regular shapes are introduced to students at this stage: triangles, circles, quadrilaterals. They offer a rich new mathematical experience in at least four ways. Children start looking for such shapes in nature, all around them, and thereby discover many symmetries and acquire a sense of aesthetics. Secondly, they learn how many seemingly irregular shapes can be approximated by regular ones, which becomes an important technique in science. Thirdly, they start comprehending the idea of space: for instance, that a circle is a path or boundary which separates the space inside the circle from that outside it. Fourthly, they start associating numbers with shapes, like area, perimeter etc, and this technique of quantization, or arithmetization, is of great importance. This also suggests that mensuration is best when integrated with geometry. An informal introduction to geometry is possible using a range of activities like paper folding and dissection, and exploring ideas of symmetry and transformation. Observing geometrical properties and inferring geometrical truth is the main objective here. Formal proofs can wait for a later stage.

Data handling, representation and visualization are important mathematical skills which can be taught at this stage. They can be of immense use as “life skills”. Students can learn to appreciate how railway time tables, directories and calendars organize information compactly. Data handling should be suitably introduced as tools to understand process, represent and interpret day-to-day data. Use of graphical representations of data can be encouraged. Formal techniques for drawing linear graphs can be taught. Visual Learning fosters understanding, organization, and imagination. Instead of emphasizing only two-column proofs, students should also be given opportunities to justify their own conclusions with less formal, but nonetheless convincing, arguments. Students' spatial reasoning and visualization skills should be enhanced. The study of geometry should make full use of all available technology. A student when given visual scope to learning remembers pictures, diagrams, flowcharts, formulas, and procedures.

1.6.2 Curricular perspectives at secondary stage in mathematics

It is at this stage that Mathematics comes to the student as an academic discipline. In a sense, at the elementary stage, mathematics education is (or ought to be) guided more by the logic of children's psychology of learning rather than the logic of mathematics. But at the secondary stage, the student begins to perceive the structure of mathematics. For this, the notions of argumentation and proof become central to curriculum now. Mathematical terminology is highly stylised, selfconscious and rigorous. The student begins to feel comfortable and at ease with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions using only terms defined earlier, and proofs justifying propositions. The student appreciates how an edifice is built up, arguments constructed using propositions justified earlier, to prove a theorem, which in turn is used in proving more. For long, geometry and trigonometry have wisely been regarded as the arena wherein students can learn to appreciate this structure best. In the elementary stage, if students have learnt many shapes and know how to associate quantities and formulas with them, here they start reasoning about these shapes using the defined quantities and formulas.

Algebra, introduced earlier, is developed at some length at this stage. Facility with algebraic manipulation is essential, not only for applications of mathematics, but also internally in mathematics. Proofs in geometry and trigonometry show the usefulness of algebraic machinery. It is important to ensure that students learn to geometrically visualise what they accomplish algebraically.

A substantial part of the secondary mathematics curriculum can be devoted to consolidation. This can be and needs to be done in many ways. Firstly, the student needs to integrate the many techniques of mathematics she has learnt into a problem solving ability. For instance, this implies a need for posing problems to students which involve more than one content area: algebra and trigonometry, geometry and mensuration, and so on. Secondly, mathematics is used in the physical and social sciences, and making the connections explicit can inspire students immensely. Thirdly, mathematical modelling, data analysis and interpretation, taught at this stage, can consolidate a high level of literacy. For instance, consider an environment related project, where the student has to set up a simple linear approximation and model a phenomenon, solve it, visualise the solution, and deduce a property of the modelled system. The consolidated learning from such an activity builds a responsible citizen, who can later intuitively analyse information available in the media and contribute to democratic decision making.

At the secondary stage, a special emphasis on experimentation and exploration may be worthwhile. Mathematics laboratories are a recent phenomenon, which hopefully will expand considerably in future. Activities in practical mathematics help students immensely in visualisation. Indeed, Singh, Avtar and Singh offer excellent suggestions for activities at all stages. Periodic systematic evaluation of the impact of such laboratories and activities will help in planning strategies for scaling up these attempts.

1.7 Let us sum up

So the above discussion of the unit has tried to give a view of vision, goals, aims and objectives of teaching and learning mathematics. It has tried to derive the educational objectives both as general objectives at the entire school level and then for the different stages of school education. Again it has tried to give a brief discussion on Bloom's taxonomy of educational objective and how there is a revised taxonomy of educational objectives justifies the current demand of mathematics education. It has given then further instructional objectives with appropriate specifications of desired students' outcomes. It has list out in detail the different behaviourial outcomes in the students while achieving the desired objectives. It has also been presented here some of the examples derived from different topic in terms of its educational objectives. Further it tries to relate the educational values of mathematics teaching with brief description on its utilitarian, cultural, disciplinary, social and recreational values. At last it has also been discussed here the different curricular perspectives of school mathematics at elementary and secondary stage.

1.8 Check your progress

Now just try to go through the following questions and check your progress.

Q.1 Differentiate between the aims and objectives of teaching mathematics?

Q.2 What are the aims and objectives of teaching mathematics at the entire school stage?

Q.3 Present a description on Bloom's Taxonomy of educational objectives and also on its revised form ?

Q.4 Write the objectives of teaching mathematics at the elementary stage?

Q.5 Write the instructional objectives of teaching mathematics at secondary stage?

Q.6 Discuss the educational values of teaching mathematics in school?

Q.7 What are the curricular perspectives of teaching mathematics at the elementary stage?

Q.8 Explain the advantages of writing instructional objectives in behavioural terms. Illustate with examples, the role of actions verbs in writing behavioural objectives. Write down five behavioural objectives each for remembering, understanding and applying of selected topics in mathematics ?

1.9 Suggested readings

NCERT;***Content-Cum-Methodology of Teaching Mathematics;*** NCERT, New Delhi (India).

N.C.T.M. ; ***The Growth of Mathematical Ideas,*** Grade K-12, 24th Year Book; Washington, USA.

IGNOU; **Nature, Objectives and approaches to teaching Mathematics;** IGNOU, New Delhi

NCERT; **National Focus Group on Teaching of Mathematics;** NCERT, New Delhi (India).

National Research Council ; ***Reshaping School Mathematics: A Philosophy and Framework for Curriculum*.** Washington, D.C.: National Academy Press, 1990

NCERT; **National Curriculum Framework;** NCERT, New Delhi (India).

Shorser, L. **Bloom's Taxonomy Interpreted for Mathematics:** www.coun.uvic.ca/learning/exams/blooms-taxonomy.html

Sidhu, K.S. **The teaching of Mathematics;** SterlingPub. New Delhi.

UNIT-III

Structure

1.0 Objectives

At the end of this unit, you will be able to:

- Understand the different aspects of methods of mathematics teaching.
- Differentiate between methods of teaching of mathematics
- Acquire a clear perspective of nature and scope of different methods of mathematics teaching
- Explain the core skills of different methods of mathematics teaching
- Enumerate the merits and demerits of different methods of mathematics teaching
- Compare and contrast different methods of mathematics teaching
- Analyse the similarity among different methods of mathematics teaching.
- Acquires an understanding on suitability of each of the methods and concerned situations.
- See the relevance of different methods with the different topics of mathematics

1.1 Introduction

The main aim of teaching is to bring about socially desirable behavior change in the students and this can only be achieved if the teaching is effective and based on the principles of teaching. How the pupils will learn effectively, depends on the methods the teacher adopts. There is the great world outside and the mind within and it is the duty of the teacher to bring the two together. This process of interpreting the world of knowledge to the child's mind is called the 'Method of teaching'. It is just a way of teaching. Method is the style of presentation of the content to the classroom.

Though teaching is an art and there are some born teachers, a majority of teachers, who have no inherent flair for teaching and are unable to arouse that much interest in the students to learn, can improve upon by practice and by following the various methods of teaching devised from time to time. So it is following the various methods of teaching devised from time to time. So it is essential that every teacher should be acquainted with different methods of teaching, which have been discussed below. It is, however important to note that a method should not become an end itself but should be used as a means to achieve the set aims of teaching. Again, the same method should not be strictly followed at all the times but should be made flexible to suit the infinite variety of circumstances and conditions existing in a given situations. The teacher is free to use a variety of the methods according to his own abilities, interest and experiences, and also of the students working under particular circumstances. A 'method' best for one teacher and applicable for a class under same circumstances may totally be a failure for another teacher to teach the same or other class under the same or different circumstances. However, there are some set criteria for the selection of a method of teaching which will be discussed later in this unit. We shall now deal with some of the commonly used methods of teaching mathematics.

1.2 Methods of teaching mathematics

'HOW TO TEACH' is really a difficult problem for the teacher. Teaching, as it is generally said is an art. Methods are the way to understand and practise the art. 'Why' and 'what' of mathematics have so far been discussed, and this unit deals with the 'how' of mathematics. "How to impart its knowledge? How to enable the child to learn, it" are the questions to be answered in this discussion. It is the final step of the execution of what we plan to teach in mathematics.

Different methods of teaching have been proposed and propounded by different educational thinkers or schools of thought in education. It is but desirable for the student to know about all of them, so that he can make a rational choice for himself. The knowledge of procedures, merits and demerits of all the methods will broaden the outlook of a would-be-teacher. The choice for him is not to be made narrow. It should be then left for him to decide from his wide information, which of the methods to use and when. The following are some of the methods of teaching mathematics.

1.2.1 Lecture Method

Lecture is a teaching method where a teacher is the central focus of information transfer. Typically, a teacher will stand before a class and present information for the students to learn. Sometimes, they will write on a board or use an overhead projector to provide visuals for students. Students are expected to take notes while listening to the lecture. Usually, very little exchange occurs between the instructor and the students during a lecture.

Merits of Lecture as a Teaching Method:

- Lectures are a straightforward way to impart knowledge to students quickly.
- Instructors also have a greater control over what is being taught in the classroom because they are the sole source of information.
- Students who are auditory learners find that lectures appeal to their learning style.
- Logistically, a lecture is often easier to create than other methods of instruction.
- Lecture is a method familiar to most teachers because it was typically the way they were taught.
- Because most teachers by default follow lecture-based methods, students gain experience in this predominant instructional delivery method.
- It provides an economical and efficient method for delivering substantial amounts of information to large numbers of student.
- It affords a necessary framework or overview for subsequent learning, e.g., reading assignments, small group activities, discussion.
- It offers current information (more up to date than most texts) from many sources.
- It provides a summary or synthesis of information from different sources.
- It creates interest in a subject as lecturers transmit enthusiasm about their discipline.

Demerits of Lecture as a Teaching Method:

- Students strong in learning styles other than auditory learning will have a harder time being engaged by lectures.
- Students who are weak in note-taking skills will have trouble understanding what they should remember from lectures.
- Students can find lectures boring causing them to lose interest.
- Students may not feel that they are able to ask questions as they arise during lectures.

- Teachers may not get a real feel for how much students are understanding because there is not that much opportunity for exchanges during lectures.
- It does not afford the instructor with ways to provide students with individual feedback.
- It is difficult to adapt to individual learning differences.
- It may fail to promote active learning unless other teaching strategies, such as questioning and problem-solving activities, are incorporated into the lecture.
- It does not promote independent learning.

Hints for a successful lecture include the following:

- Present an outline of the lecture (use the blackboard, overhead transparency or handout) and refer to it as you move from point to point.
- Repeat points in several different ways. Include examples and concrete ideas.
- Use short sentences.
- Stress important points (through your tone or explicit comments).
- Pause to give listeners time to think and write.
- Use lectures to complement, not simply repeat, the text.
- Learn students' names and make contact with them during the lecture.
- Avoid racing through the last part of the lecture. This is a common error made by instructors wishing to cram too much information into the allotted time.
- Schedule time for discussion in the same or separate class periods as the lecture.
- PREPARE. Preparation reduces stress, frustration, insecurity and consequent ineffectiveness.

Lectures are one tool in a teacher's arsenal of teaching methods. Just as with all the other tools, it should only be used when most appropriate. Instruction should be varied from day to day to help reach the most students possible. Teachers should be cautioned that before heading into numerous classes full of nothing but lectures, they need to provide their students with note taking skills. Only by helping students understand verbal clues and learn methods of organizing and taking notes will they truly help them become successful and get the most out of lectures.

1.2.2 Inductive and Deductive Method

Students have different intellectual capacities and learning styles that favour or hinder knowledge accumulation. As a result, teachers are interested in ways to effectively cause students to understand better and learn. Teachers want to bring about better understanding of the material he/she wants to communicate. It is the responsibility of the educational institutions and teachers to seek more effective ways of teaching in order to meet individual's and society's expectations from education. Improving teaching methods may help an institution meet its goal of achieving improved learning outcomes.

Teaching methods can either be inductive or deductive or some combination of the two. The inductive teaching method or process goes from the specific to the general and may be based on specific experiments or experimental learning exercises. Deductive teaching method progresses from general concept to the specific use or application. These methods are used particularly in reasoning i.e. logic and problem solving. To reason is to draw inferences appropriate to the situation. Inferences are classified as either deductive or inductive.

For example, "Ram must be in either the museum or in the cafeteria." He is not in the cafeteria; therefore he is must be in the museum. This is deductive reasoning.

As an example of inductive reasoning, we have, "Previous accidents of this sort were caused by instrument failure, and therefore, this accident was caused by instrument failure.

The most significant difference between these forms of reasoning is that in the deductive case the truth of the premises (conditions) guarantees the truth of the conclusion, whereas in the inductive case, the truth of the premises lends support to the conclusion without giving absolute assurance. Inductive arguments intend to support their conclusions only to some degree; the premises do not necessitate the conclusion.

Inductive reasoning is common in science, where data is collected and tentative models are developed to describe and predict future behaviour, until the appearance of the anomalous data forces the model to be revised.

Deductive reasoning is common in mathematics and logic, where elaborate structures of irrefutable theorems are built up from a small set of basic axioms and rules. However examples exist where teaching by inductive method bears fruit.

INDUCTIVE METHOD

Inductive approach is advocated by Pestalaozzi and Francis Bacon. Inductive approach is based on the process of induction. In this we first take a few examples and greater than generalize. It is a method of constructing a formula with the help of a sufficient number of concrete examples. Induction means to provide a universal truth by showing, that if it is true for a particular case. It is true for all such cases. Inductive approach is psychological in nature. The children follow the subject matter with great interest and understanding. This method is more useful in arithmetic teaching and learning.

Inductive approach proceeds from

- Particular cases to general rules of formulae
- Concrete instance to abstract rules
- Known to unknown
- Simple to complex

- **Steps in inductive approach**

Following steps are used while teaching by this method:-

(a) Presentation of Examples

In this step teacher presents many examples of same type and solutions of those specific examples are obtained with the help of the student.

(b) Observation

After getting the solution, the students observe these and try to reach to some conclusion.

(c) Generalization

After observation the examples presented, the teacher and children decide some common formulae, principle or law by logical mutual discussion.

(d) Testing and verification

After deciding some common formula, principle or law, children test and verify the law with the help of other examples. In this way children logically attain the knowledge of inductive method by following above given steps.

Example 1:

Square of an odd number is odd and square of an even number is even.

Solution:

Particular concept:

$1^2 = 1$	$3^2 = 9$	$5^2 = 25$	Equation 1
$2^2 = 4$	$4^2 = 16$	$6^2 = 36$	Equation 2

General concept:

From equation 1 and 2, we get

Square of an odd number is odd

Square of an even number is even.

Example 2 :

Sum of two odd numbers is even

Solution:

Particular concept:

1+1=2
1+3=4
1+5=6
3+5=8

General concept:

In the above we conclude that sum of two odd numbers is even

Example 3 :

Law of indices $a^m \times a^n = a^{m+n}$

Solution:

We have to start with $a^2 \times a^3 = (a \times a) \times (a \times a \times a)$

$$= a^5$$
$$= a^{2+3}$$
$$a^3 \times a^4 = (a \times a \times a) \times (a \times a \times a \times a)$$
$$= a^7$$
$$= a^{3+4}$$

Therefore $a^m \times a^n = (axax\ldots m \text{ times})x(axa \ldots n \text{ times})$

$$a^m \times a^n = a^{m+n}$$

MERITS

- It enhances self confident
- It is a psychological method.
- It is a meaningful learning
- It is a scientific method
- It develops scientific attitude.
- It develops the habit of intelligent hard work.
- It helps in understanding because the student knows how a particular formula has been framed.
- Since it is a logical method so it suits teaching of mathematics.
- It is a natural method of making discoveries, majority of discoveries have been made inductively.
- It does not burden the mind. Formula becomes easy to remember.

- This method is found to be suitable in the beginning stages. All teaching in mathematics is conductive in the beginning.

DEMERITS

- Certain complex and complicated formula cannot be generated so this method is limited in range and not suitable for all topics.
- It is time consuming and laborious method
- It is length.
- It's application is limited to very few topics
- It is not suitable for higher class
- Inductive reasoning is not absolutely conclusive because the generalization made with the help of a few specific examples may not hold good in all cases.

Applicability of inductive method

Inductive approach is most suitable where

- Rules are to be formulated
- Definitions are be formulated
- Formulae are to be derived
- Generalizations or law are to be arrived at.

DEDUCTIVE METHOD

Deductive method is based on deduction. In this approach we proceed from general to particular and from abstract and concrete. At first the rules are given and then students are asked to apply these rules to solve more problems. This approach is mainly used in Algebra, Geometry and Trigonometry because different relations, laws and formulae are used in these sub branches of mathematics. In this approach, help is taken from assumptions, postulates and axioms of mathematics. It is used for teaching mathematics in higher classes.

Deductive approach proceeds form

- General rule to specific instances
- Unknown to know
- Abstract rule to concrete instance
- Complex to simple

Steps in deductive approach

Deductive approach of teaching follows the steps given below for effective teaching

- Clear recognition of the problem

- Search for a tentative hypothesis
- Formulating of a tentative hypothesis
- Verification

Example 1:

Find a^2 X a^{10} = ?

Solution:

General : a^m X a^n = a^{m+n}

Particular: a^2 X a^{10} = a^{2+10} = a^{12}

Example 2:

Find $(102)^2$ = ?

Solution:

General: $(a+b)^2 = a^2+b^2+2ab$

Particular: $(100+2)^2 = 100^2 + 2^2 + (2 \times 100 \times 2)$

$= 10000+4+400= 10404$

MERITS

- It is short and time saving method.
- It is suitable for all topics.
- This method is useful for revision and drill work
- There is use of learner's memory
- It is very simple method
- It helps all types of learners
- It provides sufficient practice in the application of various mathematical formulae and rules.
- The speed and efficiency increase by the use of this method.
- Probability in induction gets converted into certainty by this method.

DEMERITS

- It is not a psychological method.
- It is not easy to understand
- It taxes the pupil's mind.
- It does not impart any training is scientific method
- It is not suitable for beginners.

- It encourages cramming.
- It puts more emphasis on memory.
- Students are only passive listeners.
- It is not found quite suitable for the development of thinking, reasoning, and discovery.

Applicability of Deductive Approach

Deductive approach is suitable for giving practice to the student in applying the formula or principles or generalization which has been already arrived at. This method is very useful for fixation and retention of facts and rules as at provides adequate drill and practice.

COMPARISON OF INDUCTIVE AND DEDUCTIVE APPROACHES

INDUCTIVE APPROACH	DEDUCTIVE APPROACH
Base: Inductive reasoning	***Base:*** Deductive reasoning
Proceeds from: Particular to general Concrete to abstract	***Proceeds from:*** General to particular Abstract to concrete
Method: A psychological method A method of discovery and stimulates intellectual powers	***Method:*** An unpsychological method A method of presentation and does not develop originality and creativity.
Learning: Emphasis is on reasoning. Encourages meaningful learning	***Learning:*** Emphasis is on memory Encourages rote learning.
Level: Most suitable for initial stages of learning	***Level:*** Suitable for practice and application
Class: Suitable for lower classes	***Class:*** Most suitable for higher classes
Participation: Enhances active participation of the students	***Participation:*** Makes the student passive recipient of knowledge
Time: Lengthy, time consuming and laborious	***Time:*** Short, concise and elegant
Facilitates discovery of rules and generalizations	Enhances speed, skill and efficiency in solving problems

Induction and deduction are not opposite modes of thought. There can be no induction without deduction and no deduction without induction. Inductive approach is a method for establishing rules and generalization and deriving formulae, whereas deductive approach is a method of applying the deduced results and for improving skill and efficiency in solving problems. Hence a combination of both inductive and deductive approach is known as "inducto-deductive approach" is most effective for realizing the desired goals.

To conclude, we can say that inductive method is a predecessor of deductive method. Any loss of time due to slowness of this method is made up through the quick and time saving process of deduction. Deduction is a process particularly suitable for a final statement and induction is most suitable for exploration of new fields. Probability in induction is raised to certainty in deduction. The happy combination of the two is most appropriate and desirable.

1.2.3 Analytic and Synthetic Method

ANALYTICAL METHOD

The word “analytic” is derived from the word “analysis” which means “breaking up” or resolving a thing into its constituent elements. The original meaning of the word analysis is to unloose or to separate things that are together. In this method we **break up** the unknown problem into simpler parts and then see how these can be recombined to find the solution. So we start with what is to be found out and then think of further steps or possibilities the may connect the unknown built the known and find out the desired result. It is believed that all the highest intellectual performance of the mind is Analysis.

- It is derived from the word analysis, its means breaking up.
- It leads to conclusion to hypothesis
- It leads to unknown to known
- It leads to abstract to concrete

Example:

if $a^2+b^2=7ab$ prove that $2\log(a+b) = 2\log 3+\log a+\log b$

Proof:

To prove this using analytic method, begin from the unknown.

The unknown is $2\log(a+b) = 2\log 3+\log a+\log b$

Now, $2\log(a+b) = 2\log 3+ \log a+ \log b$ is true

If $\log(a+b)^2 = \log 3^2 + \log a + \log b$ is true

If $\log(a+b)^2 = \log 9 + \log ab$ is true

If $\log(a+b)^2 = \log 9ab$ is true

If $(a+b)^2 = 9ab$ is true

if $a^2+b^2=7ab$ which is known and true

Thus if $a^2+b^2= 7ab$ prove that $2\log(a+b) = 2\log 3+\log a+\log b$

Merits

- It develops the power of thinking and reasoning
- It develops originality and creativity amongst the students.
- It helps in a clear understanding of the subject because the students have to go thorough the whole process themselves.
- There is least home work
- Students participation is maximum
- It this method student's participation is encouraged.
- It is a psychological method.
- No cramming is required in this method.
- Teaching by this method, teacher carries the class with him.
- It develops self-confidence and self reliant in the pupil.
- Knowledge gained by this method is more solid and durable.
- It is based on heuristic method.

Demerits

- It is time consuming and lengthy method, so it is uneconomical.
- In it, facts are not presented in a neat and systematic order.
- This method is not suitable for all the topics in mathematics.
- This does not find favour with all the students because below average students fail to follow this method.
- Every teacher cannot use this method successfully

So this method is particularly suitable for teaching of Arithmetic, algebra and Geometry as it analyses the problem into sub-parts and various parts are reorganized and the already learnt facts are used to connect the known with unknown. It puts more stress on reasoning and development of power of reasoning is one of the major aims of teaching of mathematics.

SYNTHETIC METHOD

In this method we proceed from known to unknown. Synthetic is derived form the word "synthesis". Synthesis is the complement of analysis.

To synthesis is to combine the elements to produce something new. Actually it is reverse of analytic method. In this method we proceed "from know to unknown." So in it we combine together a number of facts, perform certain mathematical operations and arrive at a solution. That is we start with the known data and connect it with the unknown part.

- It leads to hypothesis to conclusion
- It leads to known to unknown
- It leads to concrete to abstract

Example :

if $a^2+b^2=7ab$ prove that 2log (a+b) = 2log3+loga+logb

Proof:

To prove this using synthetic method, begin from the known.

The known is $a^2+b^2= 7ab$

Adding 2ab on both sides

$a^2+b^2+2ab=7ab + 2ab$

$(a+b)^2 = 9ab$

Taking log on both sides

$\log (a+b)^2 = \log 9ab$

$2\log (a+b) = \log 9 + \log ab$

$2 \log (a+b) = \log 3^2 + \log a + \log b$

$2\log (a+b) = 2\log 3+ \log a+ \log b$

Thus if $a^2+b^2=11ab$ prove that 2log (a-b) = 2log3+loga+logb

Merits

- It saves the time and labour.
- It is short method
- It is a neat method in which we present the facts in a systematic way.
- It suits majority of students.
- It can be applied to majority of topics in teaching of mathematics.
- It glorifies the memory of the child.
- Accuracy is developed by the method

Demerits

- It is an unpsychological method.
- There is a scope for forgetting.
- It makes the students passive listeners and encourages cramming

- In this method confidence is generally lacking in the student.
- There is no scope of discovery.
- The recall of each step cannot be possible for every child.

COMPARISON OF ANALYTIC AND SYNTHETIC METHODS

ANALYTIC METHOD	SYNTHETIC METHOD
Meaning: Analysis means breaking up into components	***Meaning:*** Synthesis means combining the elements to get something new.
Leads from: Unknown to known Conclusion to hypothesis Abstract to concrete Complex to simple	***Leads from:*** Known to unknown Hypothesis to conclusion Concrete to abstract Simple to complex
Method: A method of discovery and thought A psychological method	***Method:*** A method for the presentation of discovered facts. A logical method
Time: Lengthy, laborious and time consuming	***Time:*** Short, concise and elegant.
Sequence: Valid reasons to justify every step in the sequence.	***Sequence:*** No justification for every step in the sequence.
Learning: Encourages meaningful learning.	***Learning:*** Encourages rote learning
Easy to rediscover	Once forgotten not easy to recall
Encourages: Encourages originality of thinking and reasoning	***Encourages:*** Encourages memory work
Learning: Informal and disorganized	***Learning:*** Formal, systematic ad orderly
Thinking: Process of thinking	***Thinking:*** Product of thinking
Participation: Active participation of the learner	***Participation:*** Learner is a passive listener

Though both analytic and synthetic method seems to oppose each other, they complement and support each other. Analysis leads to synthesis and synthesis makes the purpose of analysis clear and complete. The teacher while teaching can use analytic methods and can encourage the student to present them in the synthetic method. Ie. Analysis forms the beginning and synthesis follow up work.

1.2.4 Heuristic Method

The word ‘Heuristic’ has been derived from the Greek word ‘Heurisco’ which means ‘I find’ or ‘I discover’ . This method implies that the attitude of students shall be that of the discoveries and not of passive recipients of knowledge. ***Armstrong*** originally introduced this method for learning of science. This method emphasis experimentation as the teacher becomes on looker and the child tries to move a head independently without any help. This method makes the student self-reliant and independent. But the teacher should develop the heuristic attitude by making a lot of preparation. The question should be so planned that it may be possible for the students to find the solution independently by proceeding in the proper direction.

DEFINITION

According to H.E.Armstrong, “This is the method of teaching which places the pupils as far as possible in the attitude of a discoverer.”

According to westaway, “the heuristic method is intended to provide training in method. Knowledge is a secondary consideration altogether.

Example 1:

The population of a city is 50,000. The rate of growth in population is 4% p.a. what will be the population after 2 years?

Teacher	:	what we have to find out in the given question
Student	:	population after two years.
Teacher	:	how can we find it?
Student	:	first we find the population after 1 year.
Teacher	:	what is the growth of every year?
Student	:	rate of growth is 4% p.a.
Teacher	:	what will be the population in the end of first year?
Student	:	population after 1 year = 50000 + 50000x 4/100
		= 50000 + 2000 = 52000
Teacher	:	what will be the base population for second year?
Student	:	the base population of second year is 52000.
Teacher	:	how can we find the growth?
Student	:	the growth of second year = 52000 x 4 /100 = 2080
Teacher	:	what will population after two years?
Student	:	population after two years = 52000 + 2080 = 54080

Example 2:

Prove that $a^0 = 1$

Teacher	:	what is 10/5?
Student	:	2
Teacher	:	what is 5/5?
Student	:	1
Teacher	:	what is 7/7?
Student	:	1
Teacher	:	what is a/a ?
Student	:	1
Teacher	:	what is a^m/a^m ?
Student	:	1. ***Equation I***
Teacher	:	how do you get the result?
Student	:	if we divide a number by itself , we will get 1.
Teacher	:	how can you write $a^m \times 1/a^m$?
Student	:	$a^{m-m} = a^0$ ***Equation II***
Teacher	:	what do you infer?
Student	:	$a^0 = 1$

Merits

- This is a psychological method as the student learns by self-practice.
- It creates clear understanding
- It is a meaningful learning
- The student learns by doing so there is a little scope of forgetting
- It develops self-confidence, self-discipline in the students
- The students acquire command of the subject. He has clear understand and notions of the subject.
- It gives the student a sense of confidence and achievement.
- The methods make them exact and bring them closer to truth.
- It inculcates in the student the interest for the subject and also develops willingness in them.

Demerits:

- It is not suitable for lower classes as they are not independent thinkers. Discovery of a thing needs hard work, patience, concentration, reasoning and thinking powers and creative abilities.
- It is very slow method. That is time consuming method.
- It is lengthy.
- The students have to spend a lot of time to find out minor results.
- The teacher may find it difficult to finish the syllabus in time.
- It does not suit larger classes.
- It suits only hard working and original thinking teachers.
- A method is successful if well-equipped libraries, laboratories and good textbook written in heuristic lines but such facilities are lacking in our school.

Heuristic method is not quite suitable for primary classes. However, this method can be given a trail in high and higher secondary classes.

1.2.5 Project Method

Project method is of American origin and is an outcome of Dewey's philosophy or pragmatism. However, this method is developed and advocated by Dr.Kilpatrick.

- Project is a plan of action – oxford's advanced learner's dictionary
- Project is a bit of real life that has been imported into school – Ballard
- A project is a unit of wholehearted purposeful activity carried on preferably in its natural setting – Dr.Kilpatrick
- A project is a problematic act carried to completion in its most natural setting – Stevenson

Basic principles of project method

Psychological principles of learning

- Learning by doing
- Learning by living
- Children learn better through association, co-operation and activity.

Psychological laws of learning

- Law of readiness
- Law of exercise
- Law of effect

STEPS INVOLVED IN PROJECT METHOD

1) Providing / creating the situations
2) Proposing and choosing the project
3) Planning the project
4) Execution of the project
5) Evaluation of the project
6) Recording of the project.

Step 1. **Creating the situation:** The teacher creates problematic situation in front of students while creating the appropriate situation student's interest and abilities should be given due importance.

Step 2. **Proposing and choosing the project:** while choosing a problem teacher should stimulate discussions by making suggestions. The proposed project should be according to the rear need of students. The purpose of the project should be well defined and understood by the children.

Step 3. **Planning the project:** for the success of the project, planning of project is very import. The children should plan out the project under the guidance of their teacher.

Step 4. **Execution of the project:** every child should contribute actively in the execution of the project. It is the longest step in the project.

Step 5. **Evaluation of the project:** when the project is completed the teacher and the children should evaluate it jointly discussed whether the objectives of the project have been achieved or not.

Step 6. **Recording of the project:** the children maintain a complete record of the project work. While recording the project some points like how the project was planned, what discussion were made, how duties were assigned, hot it was evaluate etc. should be kept in mind.

Examples

RUNNING OF A HOSTEL MESS

It involve the following steps

Step 1. The number of hostellers will be recorded.

Step 2. The expected expenditure will be calculated.

Step 3. Expenditure on various heads will be allocated to the students.

Step 4. Budget will be prepared with the help of the class.

Step 5. The account of collections from amongst the students will be noted.

Step 6. Actual expenditure will be incurred by the students

Step 7. A chart of 'balance diet' for the hostellers will be prepared.

Step 8. The time of breakfast, lunch, tea and dinner will be fixed and notified.

Step 9. Execution of different programs stated above will be made.

Step 10. Weight of each hostel will be checked after regular intervals, and the same will be put on record.

Step 11. Punctuality in all the activities of the hostellers will be recorded.

Step 12. Evaluation of the entire program, and then it will be typed out for the information of all concerned.

Some projects for mathematics

A few projects suitable for high school mathematics are listed below

- Execution of school bank.
- Running stationary stores in the school.
- Laying out a school garden.
- Laying a road.
- Planning and estimating the construction of a house
- Planning for an annual camp
- Executing the activities of mathematics clubs
- Collection of data regarding population, death rate, birth rate etc.

Merits

- This is based on various psychological laws and principles.
- It develops self-confidence and self-discipline among the students
- It provides ample scope for training.
- It provides score for independent work and individual development.
- It promotes habits of critical thinking and encourages the students to adopt problem-solving methods.
- This method the children are active participants in the learning task.
- This is based on principle of activity, reality, effect, and learning by doing etc.
- It develops discovery attitude in the child.
- It provides self-motivation as the students themselves select plan and execute the project.

Demerits It takes more time.

- The knowledge is not acquired in a sequential and systematic manner
- It is very difficult to complete the whole syllabus by the use of this method.
- It is not economical.
- Textbooks and instructional materials are hardly available.
- The project method does not provide necessary drill and practice for the learners of the subject.
- The project method is uneconomical in terms of time and is not possible to fit into the regular time table.
- Teaching is disorganised
- This method is not suitable for a fixed curriculum.
- Syllabus cannot be completed on time using this method

Though project method provides a practical approach to learning. It is difficult to follow this method for teaching mathematics. However this method may be tried along with formal classroom teaching without disturbing the school timetable. This method leads to understanding and develops the ability to apply knowledge. The teacher has to work as a careful guide during the execution of the project.

1.2.6 Laboratory Method

This method is based on the maxim "learning by doing."

- This is an activity method and it leads the students to discover mathematics facts.
- In it we proceed from concrete to abstract.
- Laboratory method is a procedure for stimulating the activities of the students and to encourage them to make discoveries.
- This method needs a laboratory in which equipments and other useful teaching aids related to mathematics are available.
- For example, equipments related to geometry, mensuration, mathematical model, chart, balance, various figures and shapes made up of wood or hardboards, graph paper etc.

Procedure:

- Aim of The Practical Work: The teacher clearly states the aim of the practical work or experiment to be carried out by the students.
- Provided materials and instruments: The students are provided with the necessary materials and instruments.

- Provide clear instructions: Provide clear instructions as to the procedure of the experiment.
- Carry out the experiment: The students carry out the experiment.
- draw the conclusions : The students are required to draw the conclusions as per the aim of the experiment.

Example 1:

Derivation of the formula for the volume of a cone.

Aims: to derive the formula for the volume of a cone.

Materials and instruments: cone and cylinders of the same diameter and height, at lease 3 sets of varying dimensions, sawdust, water and sand.

Procedure: ask the students to do the following activity.

- Take each pair of cylinder and cone having the same diameter and height
- Note down the diameter and height
- Fill the cone with saw dust / water or sand and empty into the cylinder till the cylinder is full.
- Count the number of times the cone is emptied into the cylinder and note it down in a tabular column.
- Repeat the same experiment with the other two sets of cone and cylinder and note down the reading as before.

S.NO.	DIAMETER OF CONE / CYLINDER	HEIGHT OF CONE/ CYLINDER	NO. OF MEASURES OF CONE TO FILL THE CYLINDER
1	3 CM	5 CM	3
2	5 CM	7 CM	3
3	6 CM	10 CM	3

Drawing conclusions:

Each time, irrespective of the variations in diameter and height it takes 3 measures of cone to fill the cylinder.

Volume of cone = 1/3 volume of cylinder

But volume of cylinder = $\prod r^2 h$

Volume of cone =1/3 $\prod r^2 h$

Example 2:

Sum of three angles of a triangle is 180 degree. “How we can prove this in the laboratory.

Aims:

To prove that sum of the three angles of a triangle is equal to two right angles or 180 degree.

Materials and instruments:

Card board sheet, pencil, scale, triangle and other necessary equipments.

Procedure:

In the laboratory pupils will be given on cardboard sheet each and then they are told how to draw triangles of different sizes on it. After drawing the triangles they cut this separately with the help of scissors.

Observation:

Student will measure the angles of the triangles drawn and write these in a tabular form

Figure no.	Measure of different angles			Total Angle A +B+C
	Angle A	Angle B	Angle C	
1	90	60	30	180
2	120	30	30	180
3	60	60	60	180

Calculation: after measuring the angles of different triangles in the form of cardboard sheet. We calculate and conclude their sum.

In this way by calculating the three angles of a triangle the students will be able to conclude with inductive reasoning that the sum of three angles of a triangle is 180 degree or two right angles.

Some More Topics for Laboratory Method

Derivation of the formula for the

- Circumference of a circle, area of circle
- Area of square, rectangle,, parallelogram, and trapezium
- Area of triangle, right angled triangle, isosceles right angles triangle
- Total surface area of cone, cylinder
- Volume of a sphere
- Volume of a cone

Expansion of identities such as $(a+b)^2$, $(a-b)^2$, $(a+b+c)^2$

Verification of

- Properties of certain geometrical figures like parallelogram, rhombus etc
- Angle sum property in a triangle
- Congruency postulates
- Theorems relating to triangles, circles and transversal properties.

Merits:

- The method is based on the principle of learning by doing.

- This method is psychological as we proceed from known to unknown.
- It is based on the student's self pacing.
- It helps in making clear certain fundamental concepts, ideas etc.
- It develops the self-confidence and teaches the students the dignity of labour.
- The children learn the use of different equipments, which are used in laboratory.
- It develops in the child a habit of scientific, enquiry and investigation.
- This method presents mathematics as a practical subject.
- It stimulates the interest of the students to work with concrete material.
- It provides opportunities for social interaction and co-operation among the students.
- It is child-centred and therefore it is a psychological method.
- It helps the students to actively participate in the learning process and therefore the learning becomes more meaningful and interesting.

Demerits:

- This method can be used for a small class only.
- It requires a lot of planning and organization.
- This method is suitable only for certain topics.
- This method it is not possible to make progress quickly.
- This method requires laboratory equipped with different apparatus.
- All mathematics teachers cannot use this method effectively.
- It is an expensive method. All schools are not able to adopt this method.
- This method has very little of theoretical part in it.

In conclusion we can say that this method is suitable for teaching mathematics to lower classes as at this stage teaching is done with the help of concrete things and examples.

1.2.7 Problem Solving Method

The child is curious by nature. He wants to find out solutions of many problems, which sometimes are puzzling even to the adults. The problem solving method is one, which involves the use of the process of problem solving or reflective thinking or reasoning. Problem solving method, as the name indicated, begins with the statement of a problem that challenges the students to find a solution.

Definition

- Problem solving is a set of events in which human beings was rules to achieve some goals – **Gagne**

- Problem solving involves concept formation and discovery learning – **Ausubel**
- Problem solving is a planned attacks upon a difficulty or perplexity for the purpose of findings a satisfactory solution. – **Risk,T.M.**

Steps in Problem Solving / Procedure for Problem solving

1. Identifying and defining the problem:

The student should be able to identify and clearly define the problem. The problem that has been identified should be interesting challenging and motivating for the students to participate in exploring.

2. Analysing the problem:

The problem should be carefully analysed as to what is given and what is to be find out. Given facts must be identified and expressed, if necessary in symbolic form.

3. Formulating tentative hypothesis

Formulating of hypothesis means preparation of a list of possible reasons of the occurrence of the problem. Formulating of hypothesis develops thinking and reasoning powers of the child. The focus at this stage is on hypothesizing – searching for the tentative solution to the problem.

4. Testing the hypothesis:

Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternate hypothesis and proceed.

5. Verifying of the result or checking the result:

No conclusion should be accepted without being properly verified. At this step the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalizations and apply it to their daily life.

Example :

Define union of two sets. If $A=\{2,3,5\}$. $B=\{3,5,6\}$ And $C=\{4,6,8,9\}$.

Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Solution :

Step 1: Identifying and Defining the Problem

After selecting and understanding the problem the child will be able to define the problem in his own words that

(i) The union of two sets A and B is the set, which contains all the members of a set A and all the members of a set B.

(ii) The union of two set A and B is express as '$A \cup B$' and symbolically represented as $A \cup B = \{x \,;\, x \in A \text{ or } x \in B\}$

(iii) The common elements are taken only once in the union of two sets

Step 2: Analysing the Problem

After defining the problem in his own words, the child will analyse the given problem that how the problem can be solved?

Step 3 : Formulating Tentative Hypothesis

After analysing the various aspects of the problem he will be able to make hypothesis that first of all he should calculate the union of sets B and C i.e. $(B \cup C)$. Then the union of set A and $B \cup C$. thus he can get the value of $A \cup (B \cup C)$. Similarly he can solve $(A \cup B) \cup C$

Step 4: Testing Hypothesis

Thus on the basis of given data, the child will be able to solve the problem in the following manner

In the example it is given that

$$\begin{aligned} B \cup C &= \{3,5,6\} \cup \{4,6,8,9\} \\ &= \{3,4,5,6,8,9\} \\ A \cup (B \cup C) &= \{2,3,5\} \cup \{3,4,5,6,8,9\} \\ &= \{2,3,4,5,6,8,9\} \end{aligned}$$

Similarly,

$$\begin{aligned} A \cup B &= \{2,3,5,6\} \\ (A \cup B) \cup C &= \{2,3,4,5,6,8,9\} \end{aligned}$$

After solving the problem the child will analyse the result on the basis of given data and verify his hypothesis whether $A \cup (B \cup C)$ is equals to $(A \cup B) \cup C$ or not.

Step 5 : Verifying of the result

After testing and verifying his hypothesis the child will be able to conclude that $A \cup (B \cup C) = (A \cup B) \cup C$

Thus the child generalises the results and apply his knowledge in new situations.

Merits

- This method is psychological and scientific in nature
- It helps in developing good study habits and reasoning powers.
- It helps to improve and apply knowledge and experience.
- This method stimulates thinking of the child
- It helps to develop the power of expression of the child.

- The child learns how to act in new situation.
- It develops group feeling while working together.
- Teachers become familiar with his pupils.
- It develops analytical, critical and generalization abilities of the child.
- This method helps in maintaining discipline in the class.

Demerits

- This is not suitable for lower classes
- There is lack of suitable books and references for children.
- It is not economical. It is wastage of time and energy.
- Teachers find it difficult to cover the prescribed syllabus.
- To follow this method talented teacher are required.
- There is always doubt of drawing wrong conclusions.
- Mental activities are more emphasized as compared to physical activities.

Problem solving method can be an effective method for teaching mathematics in the hands of an able and resourceful teacher of mathematics.

1.3 Let us sum up

A number of methods of teaching have been discussed. Some of them have been recommended for use, some have been disapproved and some have been recommended for use with caution. Out of all the available methods, every teacher has to make his own choice. This choice cannot be whimsical choice, but it will have to be made in a rational way keeping in view the facilities available and the nature of the work to be done. Certain methods have been fully approved whereas anothers have been partially approved. There are still others which have been outrightly approved for a given learning situations. After understanding all of them the teachers have to make their own decision as to which of the methods is best for them. Each of the methods like inductive-deductive, analytic-synthetic, problem-solving, and heuristic can be reasonably supported to be one of the most suitable methods. It is no way difficult to argue in support of them, but it cannot be always possible for the teacher to use the above methods in all the conditions, situations and topics. There are some of the topics in mathematics which can be taught quite effectively through lecture methods too. Sometimes the heavy load of curriculum also justifies the use of lecture methods. Sometimes it is also possible to mix two or more methods too to make the teaching become more effective. It is always true to be inductive in the approach of teaching as its always leads for the betterment of the students understanding.

In the end, we can safely conclude that it is wrong to name one single method as the best method. A good teacher will so digest or absorb them all that he evolves his own method comprising good points of all the methods. He will not permit any of the methods to become his master but will remain a true master of all of them.

1.4 Check your progress

Now just try to go through the following questions and check your progress.

Q.1 Which is the best method of teaching mathematics according to your opinion. Discuss?

Q.2 Illustrate with examples the Inductive-Deductive method of teaching mathematics?

Q.3 What are the benefits of project method of teaching mathematics?

Q.4 A teacher in practical cannot exclusively employ a single method in teaching of mathematics and employs a combination of methods. Discuss?

Q.5 How will you employ Analytic-Synthetic method of teaching mathematics?

Q.6 What is the most traditional way of teaching mathematics in school. Write its drawbacks and suggest a mechanism to overcome them ?

1.5 Suggested readings

Carpenter, T. P. (1989). 'Teaching as problem solving'.

NCERT;***Content-Cum-Methodology of Teaching Mathematics;*** NCERT, New Delhi (India).

N.C.T.M. ; ***The Growth of Mathematical Ideas,*** Grade K-12, 24th Year Book; Washington, USA.

IGNOU; **Nature, Objectives and approaches to teaching Mathematics;** IGNOU, New Delhi

NCERT; **National Focus Group on Teaching of Mathematics;** NCERT, New Delhi (India).

National Research Council ; ***Reshaping School Mathematics: A Philosophy and Framework for Curriculum*****.** Washington, D.C.: National Academy Press, 1990

NCERT; **National Curriculum Framework;** NCERT, New Delhi (India).

R.I.Charles and E.A. Silver (Eds), ***The Teaching and Assessing of Mathematical Problem Solving,*** (pp.187-202). USA: NCTM

Sidhu, K.S. **The teaching of Mathematics;** SterlingPub. New Delhi.

UNIT-IV

Structure

1.0 Objectives

At the end of this unit, you will be able to:

- Appreciate the role of textbooks and other reference materials in teaching of mathematics;
- Acquire the ability of using textbooks and other reference materials;
- Be able to understand the use of audio-visual aids in teaching of mathematics;
- Explain lesson planning and preparation of scheme of lessons in mathematics;
- Diagnose the learning difficulties of the learners
- Enumerate the remedial strategies for the students with learning difficulties

1.1 Introduction

This unit describes the importance of textbooks in teaching and learning of mathematics. It further describes the role of mathematics textbooks in development of different idea and skills among the students. It also describes the role of other reference materials in the teaching and learning of mathematics. Audio-visual aids also supports very much in the teaching learning processes of the students. As mathematics is thought to be very abstract subject the need of audio-visual aids is very important as it helps in understanding the mathematical concepts very clearly. This unit tries to highlight the use of audio-visual aids in teaching of mathematics.

Lesson planning is a must for the success of content delivery and to achieve the objectives of a lesson. The unit discuss then about lesson planning and preparation of scheme of lessons in mathematics.

In the last part of the unit the diagnosis of learning difficulties has been presented in brief and further remedial strategies has been discussed to address the learning difficulties in mathematics of the school going children.

1.2 Instructional Materials in Mathematics: Textbooks, work-books, guidebooks, Reference books, other sources.

Teaching methods in use today, e.g. pipils working individually or in group , often require that the pupil get mathematical meaning from the written word. In past, conventional class-teaching might use a textbook only as a source of exercise. Now, due to the growth of mixed ability grouping and individual learning systems, textual materials has become the source of explanations and instruction as well as of exercise. Every year, many student-teachers begin teaching mathematics in schools. Textbooks which are well written and suitable for the pupils will be the most appreciated gift to those inexperienced teachers.

Even in a qualified mathematics teacher's class or a conventional class-teaching class, pupils still have to learn mathematics by reading a textbook in many circumstances; e.g. some pupils may not like their mathematics teachers, so they don't want to pay attention in mathematics lessons; pupils may be absent in some mathematics classes due to illness or being involved in some other activities,etc. To catch up with the rest of the class, they might have to read the textbook by themselves.

The textbook may also play the role of authority to pupils in a a mathematics lesson. In many mathematics courses, pupils may have only one textbook, and in addition, their teachers' teaching relies heavily on that textbook, then to them the textbook is equal to the course in this case. Another situation occurs that the textbook provides answer to the questions. From different points of view, we have seen the importance of the textbook in mathematics lessons. There has been a wish to produce textbooks which pupils can read easily by themselves.

In children Reading Maths (1980,p.26), the authors state that acquisition of *concepts, principles, skills,* and *problem solving strategies* are important goals which school mathematics textbooks aim for. Besides these four items, we think *culture* and *attitude* are important too.

Concepts

Mathematics textbooks provide material to expound concepts for the reader. One of the fundamental numeric concepts in school mathematics is to know the concrete meaning of numbers, e.g. fraction, decimal, negative numbers, etc. Geometric materials tell us the concept of our living space. An algebraic material shows us many models which are the approximation of physical daily life problems. Probability materials tell us how to make reasonable prediction. Different means of a collection data tells us some concrete concepts about that data.

Principles

There are many fundamental principles in school mathematics textbooks. The principles of deduction leads us think axiomatically. The principle of mathematical induction convinces us to believe some facts are true for any natural number. To find the volume of a solid, we apply Cavelieri's principle. To enumerate the ways of arranging subjects we apply the principle of addition and multiplication.

Skills

Skills mean the methods of solving problems. For instance, Euclidian algorithm, solving equations, method of approximation and method of translation are powerful tools for later work and study.

Problem-Solving Strategies

Mathematics textbooks serve to develop the capacity of the human mind for the observation, selection, generalization, abstraction and construction of models for use in solving problems in mathematics as well as in other disciplines.

Culture

Mathematics history is a creative culture. The process of construction numbers, the background stories of discovering theories as well as the stories of some great mathematicians, such as Newton, Gauss, Euler, Cauchy, Napier, Ramanujam, Descrates, Kepler, Fermat, Abel, etc., are good source for the goal of culture.

Attitudes

During every mathematics lesson a child is not only learning, or failing to learn, mathematics as a result of the work he is doing but is also developing has attitude towards mathematics. Positive attitudes assist the learning of mathematics; negative attitudes not only inhibit learning but every often persist into adult life and affect choice of job, as Mathematics Counts says (1982, p.101).

Puzzles and projects in textbooks always encourage us to do it. Motivations in textbooks arouse our curiosity of finding out something. If the textbooks can present mathematics in such a way as to continue to be interacting, enjoyable, and inspiring then hopefully the readers will develop positive attitudes towards mathematics.

A textbook is a collection of series of texts on various concerns of a specific area, e.g., a textbook on mathematics will be consisting of various topics in mathematics specific to a particular grade and comprising specific text on these topics. A textbook of class X may differ in various aspects from a textbook at college level for the same subject. In class X textbook, ‘trigonometry’ may be one of the several chapters while there may be a complete textbook on ‘trigonometry’ alone at college level.

Textbook may vary according to the subject, content, student’s age level etc. it gives introduction of the content and then tries to inculcate the understanding of the content through various instances, fundamentals, examples, exercises, etc. a textbook should preferably have historical content too, as it sensitizes learner to humanistic aspect of mathematics. As Lawrence(2006) reveals that the historical content offers a flexible framework within which it is possible to achieve good results. It may have story also to develop communication skills, empathy, understanding and above all the subject knowledge.

From the above discussion, it can be said that a textbook is a very important learning resource as it not only introduces the content, but also builds a platform on which the entire structure of the concept and content knowledge stands firmly. It also gives a framework of a particular area of a course of study. It may be made easily available to all students irrespective of the socio-economic background. Now a days textbooks are available in hard as well as soft forms. It can be in the form of a paper book or an e-book.

Apart from the textbook, there are other types of books, such as handbooks and reference books. A handbook is a complete book in concise form on a particular task, profession, or area of study etc. A reference book may consists of details or further explanation on a particular topics of a textbook, extension of the topic, further examples/problems and further suggestive texts. Such books may be used to supplement to any resource and also to expand and strengthen the content. As an example, the book ‘The Mathematics of Egypt, Mesopotamia, China, India and Islam:

A Sourcebook' edited by Victor J Kaltz (2007) can be used as a reference book to know historical development in mathematics. In continuation of handbook , work books and reference books, and supplementary books , viz., 'Exemplar Problems in Mathematics' and 'Laboratory manuals in Mathematics' by NCERT for various classes, may also be seen as good learning resources.

1.3 Audio-visual Aids in Teaching Mathematics

The term audio visual may refer to works with a sound and a visual component, the production or use of such works, or the equipment involved in presenting such works. Audiovisual aids are defined as any device used to aid in the communication of an idea. Virtually anything can be used an aid, providing it successfully communicates the idea or information for which it is designed or to elicit a desired audience response. Modern age is the age of science and technology. It goes on changing day by day. A teacher should adopt technology in order to teach the students better in a better and effective manner. The instructor should select the aids which will be most effective in presenting the skills and knowledge that are to be gained in the lesson. The instructor must take into account, however, the limitations of the instructional aids that are available for use in the lesson.

Talking about the importance of audio visual aids in teaching of mathematics, the use of sensory aids the teaching of mathematics is of recent origin. Infact, all teaching has always involved the communication of ideas through the medium of speech, or visual by the use of written or printed material. Text books, writing materials, geometrical instruments and the chalk board (all these are sensory aids) have long been regarded as indispensable equipment for mathematics classes. For many years' resourceful teachers have use models, instruments, drawing and other devices to interest and facilitate learning. But for a long time the potential values of these supplementary devices were fully realized only by exceptional teachers. Mathematics is an essentially a subject, where doing is more prominent than reading. That is why a certain amount of equipment is indispensable in order to make even a start in this subject. Moreover, it is held by a vast majority of people that mathematics is a dry and difficult subject, full of abstract things. The result is that students take very little interest in it. To create the necessary interest is a constant problem for the teacher.

This subject demands the use of aids at every step. Equipment for mathematical instructions falls into two categories; (1) That which the students need in order to pursue his owns individual study and (2) that which can be used in common and has to be provided by the school in the mathematics laboratory. The former category includes such obvious necessities as test book, writing equipment, simple drawing and measuring instruments in the form of a geometry box, and in some cases special equipment such as the slide rule. These are an essential part of good teaching. They have many advantages asunder;

Audio video devices enhance the interest of students, especially students of quite young age. As children take interest in colors and different devices, instruments, it's quite easy to teach them. As well as teenagers also take interest in pictures etc. They also want to do their work by themselves. So audio video teaching is much effective than conservative teaching. Good audio visual aids can clarify points, present information, illustrate arguments and processes, and do a hundred useful things in a presentation.

Audio video devices make a lesson easy and interesting, help in remembering/information of habits. Some of the teacher's aids are given below.

Chalk board:

This is the first and foremost of all the items of mathematical equipment. It must be there even if anything else is not there. This is the minimum equipment. But sometime it need not even be included in the list of special equipment, as it is a taken for granted part of every classroom. Chalk board may verily be called the second tongue of the mathematics teacher.

Charts:

Charts can cover a vast range of mathematical topics, such as coins, weights and measures: prices of different kinds of articles; school and class pass percentage; different kinds of geometrical figures and their qualities; different kinds of angles, triangles, polygons, quadrilaterals: different types of bank drafts or cheques; arithmetical terminology such as fraction, average, area, profit and loss, ratio etc

Filmstrips:

Various filmstrips can be used to give a new color and attraction to different ideas of mathematics. The resourceful teacher can obtain them from the market. The advantages lie in the fact that they don't burden the mind, can be shown in off hours, and teach the subjects in an effective manner. The mathematics teacher should seek the cooperation of the science teacher in using these aids.

Radio:

Broadcasting stations can also help, if the school possesses a radio set and the stations broadcast programmes of mathematical interest .these broadcast can relate important incidents form the lives of great mathematicians .Some talk by expert can be arranged on the place of mathematics in daily life, and in industry and trades. The history of the development of mathematics can also be a topic, and special emphasis can also be ladino the discoveries made by mathematicians. Similarly news pertaining to market rates, temperature and rainfall, broadcast by radio stations can serve as source of collection of data.

Homemade equipment:

Some items can be made by the students and teachers. These pieces of equipment may not be very precise and accurate but have two notable advantages over those that are produced commercially. First student always take pride in the equipment made by them with the result that their interesting using such equipment is increased. Secondly, they are more likely to understand clearly fundamental principles upon which mathematical instruments are based. This second advantage is more important that the first. Among the instruments and items of equipment which can be made by students and teachers may be maintain the following.

- Beads, balls sticks, pebbles, number-picture cards.
- Almost all the charts.
- Portraits of great mathematicians.
- Almost all the possible models.
- Graphs, budgets etc
- Stencils for geometric figures.
- Bulletin boards and display cases.
- Black board instruments such as rulers, protectors, compasses etc

Audio Visual AIDS are materials using sight or sound to present informing. To use Audio/Visual Aids in teaching you have to bridge the gap between the different types of learners by adding audio/visual aides to your teaching techniques. Implement 'show and tell l' sessions to promote student involvement. Provide audio/visual aides to demonstrate mathematical concepts to students because this will help students learn to think of complicated material in a practical way. Watch videos and movies that reinforce lesson plans. Invite guest speakers to help students learn concepts. Technology can greatly aid the process of mathematical exploration, and clever use of such aids can help engage students. Calculators are typically seen as aiding arithmetical operations; while this is true, calculators are of much greater pedagogic value. Indeed, if one asks whether calculators should be permitted in examinations, the answer is that it is quite unnecessary for examiners to raise questions that necessitate the use of calculators. On the contrary, in a nonthreatening atmosphere, children can use calculators to study iteration of many algebraic functions. For instance, starting with an arbitrary large number and repeatedly finding the square root to see how soon the sequence converges to 1, is illuminating.

Even phenomena like chaos can be easily comprehended with such iterators. If ordinary calculators can offer such possibilities, the potential of graphing calculators and computers for mathematical exploration is far higher. However, these are expensive, and in a country where the vast majority of children cannot afford more than one notebook, such use is luxurious. It is here that governmental action, to provide appropriate alternative low-cost technology, may be appropriate. Research in this direction will be greatly beneficial to schooleducation. It must be understood that there is a spectrum of technology use in mathematics education, and calculators or computers are at one end of the spectrum. While notebooks and blackboards are the other end, use of graph paper, geo boards, abacus, geometry boxes etc. is crucial. Innovations in the design and use of such material must be encouraged so that their use makes school mathematics enjoyable and meaningful.

1.4 Lesson Planning and Preparation of Scheme of Lessons in Mathematics

- **Preparation – getting to know your class**

When planning to teach a class, knowledge of the group and of the individuals is essential. Your first contact with the groups you will teach will be through observation and you should exploit this opportunity to learn as much as you can about the class.

The following information may be useful to record when you first observe a class that you are going to teach:

- The way in which the pupils are grouped - both within the department and within the classroom i.e. Are they in ability sets or mixed ability groups? Does the pupil or the teacher choose seating position?
- The classroom entry and exit routines
- The range and type of activities the pupils are engaged in
- How the pupils' time is allocated
- How the teacher uses her/his time
- The range of organisational routines utilised throughout the lesson e.g. management of resources, teacher expectation in relation to pupils responding to questions, seeking help etc.
- How and to what extent the pupils are involved in the classroom organisation
- How the teacher controls activities
- The nature and extent of interaction between teacher/pupils and pupils/pupils
- How much planning has taken place prior to the lesson
- How the layout of the room and availability of resources determines organisation
- If there are children with special needs in the class, what their needs are and how they are supported

Also you should:

- make a plan of the classroom including pupils' normal seating arrangement
- obtain a class list
- obtain details of the mathematics scheme of work for the class, and which aspects/topics you are to teach
- make a list of the mathematics resources available/practical equipment, computers & software, video etc.
- find out at what Mathematics attainment level the pupils in the class should be working
- find out how pupils' work is marked/checked and when/how much homework is set

Unit of work planning guidelines

The mathematics section of your school will have schemes of work available for you to consult, which set out an overview of the topics to be covered by each year group, and possibly each set within it. Although these schemes are invaluable in providing you with a general order and guide, it is important to realise that when you take on the teaching of a class, you will need to think in more detail. You will need to plan the progression through each topic area carefully to take account of the pupils' previous experience, current level of knowledge and understanding and the learning outcomes you hope they will achieve by the end of the group of lessons you are planning and teaching. The unit of work plan provides information not only about what may be taught and learnt, but also gives an indication of the range of teaching and learning strategies to be employed and the resources to be used.

Units of Work form the overview of work planned for the topic you are teaching and should be produced in consultation with your mentor, class teacher and school subject tutors.

Suggested headings:

- Subject

The topic or sub-topic to be covered

- *Particulars of the class*

Numbers, gender mix, age, groupings, pupils with special educational needs, etc.

- Previous experience/knowledge of the topic

Give some idea of what you think the pupils know and can already do in relation to this topic, i.e. the knowledge, skills and conceptual understanding that you will build on. Consider how this topic links with other areas of mathematics.

- Learning objectives and learning outcomes

Learning objectives give details of what you intend pupils to learn. *Learning outcomes* describe what learning you expect to observe in order to assess that learning objectives have been achieved. They should be specific enough to help you in judging the success of a lesson in terms of learning for different pupils. You should identify learning objectives for both the starter and the main activity parts of the lesson.

- Key vocabulary / notation

Itemise the key vocabulary or specific mathematics notation to be introduced during this unit of work.

- Content - teacher explanations and pupil activities

This is a sequential summary of the content that must be tackled. It should demonstrate both continuity within the topic, and progression of development. It is better to have too much at the outset so that you can select content once you know the pupils' range of abilities better. This list needs to be matched to the learning objectives already identified.

- Organisation

Consider and record:

  - when you will use whole class teaching, group work and individual work
  - how the class will be arranged
  - what provision you will make for the slower and quicker finishers; the able and less able, pupils.

- Resources

Identify where possible for both starter and main activity sections of the lesson, the resources and activities to be utilised by teacher and pupils.

- Assessment

Indicate how you will evaluate the success of the unit in terms of pupil learning and motivation. You may use questioning or introduce a short activity into your plenary to assess what pupils have learnt.

- How will you start?

The start, at least, needs to be very detailed e.g. the minimum of the first lesson must be prepared carefully before you start teaching the unit. You may wish to prepare the second lesson also but remember that the second lesson may change in light of your experience in the first lesson. Remember, when planning your first lesson, to include a way of ascertaining if pupils do indeed have the prior knowledge and understanding that you are assuming.

Lesson planning guidelines

A lesson plan must be prepared for every lesson or learning experience, whether this is for a whole class or a small group. If you are only responsible for teaching a part of the lesson, this must also be carefully planned and checked with your teaching partner.

A lesson plan proforma is attached at the back of this guide and should be used for all lessons throughout your secondary placements.

Each lesson plan should contain:

- Class name and regular teacher's name
- Topic to be taught
- Duration of lesson
- National Curriculum targets and levels and National Strategy key objectives
- Learning objectives

  The learning objectives for the activity/lesson should be specified in realistic terms: What facts, skills, concepts, processes, mathematical language/terminology do you expect pupils to know, use and understand?

  Learning objectives should be written as:

Pupils will know that: .... (facts)

Pupils will be able to:...................................... (skills)

Pupils will develop an understanding of........... (concepts)

Pupils will know, understand and be able to use appropriately…………..

(mathematical language)

- Personal teaching objectives

  e.g. improve your management of resources, improve your questioning skills

Include these in your lesson plan as a reminder to yourself and also to inform any observer of your focus.

- Assessment

  What criteria will you use to assess learning?

  What will you consider to be evidence of learning (learning outcomes)?

How will you organise the assessment of pupils during the lesson?

How will this information inform your next lesson plan?

- Relationships to previous work

  How do the learning objectives to be addressed in this lesson relate to previous lessons in the unit or build on/relate to work previously done?

  Are you assuming any pre-requisite knowledge/skills that will affect the outcome of learning?

- Pupil misconceptions

  Are there common misconceptions associated with this work that you need to plan to reveal and address? If there are, ensure that you have planned activities/questions that will reveal them.

- Outline of lesson

  Record what you will be doing and think carefully about what you expect pupils to be doing.

  If a formally constructed lesson is being planned it should show the starter activity, introduction to main section, development of main task/activity and the conclusion. An estimation of the timing of each part is useful.

  Seating arrangements and details of pupil grouping for various parts of the lesson should be considered and recorded. (e.g. whole class; individual; pairs; group….)

- Starter

  Will you use a mental maths activity initially?

  If so, consider the focus in terms of:

  - links to previous lessons/ main activity
  - preparation for future topics or lessons
  - maintenance revision of previous work
  - timing, teaching approach & resources

- Main activity

  Introduction

  How will you start?

  Outline what pupils will be learning (share learning objectives)

Questions you might ask to promote interest or to bring out important teaching / learning points.

Introductory / key questions / explanations.

What contexts, visual aids will you utilise?

Consider the purpose, nature and organisation of board-work

What examples will be used?

Content of activity

Organisation and structure

Groupings and timings for each phase

Explanation of the task

Explanation of organisation and time allotted to pupils

Check understanding

Explain expected outcomes

Consider what demands will be made on your time

Secondary activities (alternative or extra activities to meet the needs of specific individuals) e.g. supplementary tasks for those who finish quickly, extension activities for more able, support activities/materials for less able

- Plenary/conclusion

  Note how you will:

  - give pupils sufficient notice to draw their work to a sensible conclusion
  - share what has been learnt/discovered
  - address any misconceptions
  - consolidate and summarise main learning
  - give out homework
  - collect in work resources
  - end on time

- Materials and resources

  What resources are to be used by the teacher and/or pupils?

  What examples will you show on the board (if any)?

  What tasks from the textbook or worksheet will be set? (Note: If pupils are to be given sections from a text book or work sheet to complete, you should have worked through them earlier and made notes of any awkward questions or contexts the pupils may meet.)

How will you keep track of resources (calculators, rulers, etc.)?

- Homework

  What is the purpose and content of the homework?

  What is the deadline?

  How should pupils record the instructions?

  How will you check?

Lesson evaluation guidelines

A critical evaluation of the lesson should be made as soon as possible after the lesson. A lesson evaluation proforma is attached at the back of this guide and should be used for all lessons throughout your secondary placements.

The main issues that you will need to address are:

- How do the learning outcomes relate to the learning objectives set? Were all, or only some, achieved? Consider this question from the point of individuals as well as a whole class perspective.
- If learning objectives were not achieved, consider why this was so.
- How do the answers to the above affect the planning for the next lesson?
- Were the personal performance objectives achieved?

The following are questions which may offer foci for your evaluation and reflection. Remember to select one or two which are pertinent to your current targets and action points.

- General questions
- How closely did you stick to your plan?
- If you needed to deviate from the plan - how, why? To what extent may this affect future lesson plans with this class?

- Did any part of your teaching go particularly well? Can you think of a reason for this? Will this affect future planning?
- Did any part of your lesson go particularly badly? Can you think of a reason for this? Will this affect future planning?
- Which aspects could be improved? How?

➢ What did the pupils learn?
(Facts, skills, concepts, processes, specialist language)

- Which aspects of the learning did pupils find difficult or not achieve?
- Did any particular pupil / group of pupils experience difficulty - why?
- How will you address this?

➢ Suitability / match of the work

- Were the more able pupils challenged?
- Could the less able pupils cope? (Were they challenged and yet given an opportunity to be successful?)
- Were pupils interested and motivated?
- Were pupils actively engaged in the lesson? If so - can you analyse why and plan to repeat? If not, what will you do to ensure they are interested next lesson?
- Was there any misbehaviour? Why? Could it be due to poorly matched work or boredom? If not this, then what?
- Did the reactions of any particular children strike you? If so, why? What can you learn from this?

➢ What else must you take into account when planning future sessions?

- Class organisation – grouping, seating, class routines, expectations etc.
- Did the lesson flow smoothly? If not, what was the cause, how can it be addressed in future?
- Could the pupils hear you?
- Was your language appropriate?
- Were your explanations clear?
- Were the resources appropriate and readily available?
- Were the instructions (verbal or written) clear?
- Were the tasks interesting and appropriate for the learning ?
- Was the pupil grouping successful and appropriate for the task?

- Were you at any point surprised by anything going on in the class? Do you need to adjust your future planning to take account of it?

➢ Questions

- Did the children ask questions? If so, was this because they were interested or because they did not understand?
- When you asked questions of the whole class, what was the response? Was the response what you expected or hoped for?
- If you did not get responses, or only from a limited number, why do you think this was so? What can you do to improve the spread of responses?
- What proportion of the pupils asked or responded to questions? Did you attempt to differentiate your questions in relation to the pupils present?
- How effective was your questioning technique? Did you use both open and closed questions?

➢ Contact with individuals

- Which pupils do you think received most attention from you today? Why?
- Are you aware of any pupils who do not get your attention on a regular basis? What strategies can you use to counteract this?
- Did you praise any pupils today? Why?
- Did you tell off any pupils today? Why?

➢ Teaching content

- Did you have any difficulty in achieving your objectives? If so, why?
- Did you feel you achieved anything not stated in your objectives?
- Was the lesson appropriate in content, level and pace?
- Were progressions within the lesson and from previous lessons developmental and logical?
-

➢ Future plans

How will the way this lesson went affect future lessons in terms of:

- learning outcomes for pupils
- classroom organisation
- lesson content
- nature of learning tasks
- your own teaching approach and behaviour ?

1.4.1 A format Lesson Plan in Mathematics

Topic	Student teacher’s particular
Class particulars	
Previous experience	
Learning objectives	
Learning outcomes	
Key vocabulary and notation	

Starter	Main Teaching	Notes	Plenary

LESSON PLAN		
TEACHER	**CLASS**	
DATE	**PERIOD**	**DURATION**
TOPIC		
STUDENT TEACHER PARTICULARS		
INTENDED PUPIL LEARNING OBJECTIVES (differentiated as required)		
ASSESSMENT of LEARNING (How will you know what learning has taken place?)		

Timing	Aspect of Lesson	Teacher input, questions, explanations, resources; Pupil activities, resources, organisation
	Starter	
	Main Section **Introduction** Introductory questions Relationship to previous lesson Reminder of outcomes Check of understanding Explain organisation & timing	

Timing	Aspect of Lesson	Teacher input, questions, explanations, resources; Pupil activities, resources, organisation
	Main Section **Main Activity** Introduction- key points/questions Content Organisation & structure Groupings / timings Questions to promote interest Main teaching points	
	Support **Activities**	
	Extension **Activities**	

Timing	Aspect of Lesson	Teacher input, questions, explanations, resources; Pupil activities, resources, organisation
	Plenary Consolidate & summarise main learning Address any errors/misconceptions Share work, feedback & praise	
	Closing Give notice for end of session Collect in work **Plan tidying/ending on time**	

Homework
Resources Equipment Materials
LESSON EVALUATION
Did I follow the plan?   **If the plan changed, how and why?**   **What went well?** **What aspects could be improved?**
What did the pupils learn? How do I know? Facts Skills Concepts Attitudes Discipline Personal Qualities

What aspects of the learning did pupils find difficult or not achieve?
What did I find out about the pupils and their learning to take into account in the future
What else must I take into account when planning future sessions? Organisation Lesson content Presentation Resources Timing Discipline
Other comments (refer to some of the questions suggested in the guide)

1.5 Diagnosis of learning difficulties in Mathematics and Remedial measures

The aims and objectives of the Primary School Curriculum, Mathematics are valid for all students. However, not all students learn mathematics in an even and predictable manner. The abstract and conceptual nature of mathematics poses particular challenges to students with average general learning abilities. The teacher has a pivotal role in mediating the objectives of the School Curriculum, To meet the needs and of all the students with learning diffuculties, greater emphasis is placed on the social, rather than the creative and aesthetic value of mathematics without excluding those important aspects. There will be particular emphasis on managing money, understanding timetables and using measures in everyday life situations. The acquisition of these skills must be prioritised in order to equip the student to participate fully and independently in society.

A level of proficiency in basic mathematics is needed to cope independently and effectively with everyday living including telling the time, shopping, reading timetables, cooking, measuring, and so forth. These guidelines aim to provide teachers with an understanding of the particular barriers to learning mathematics that students with learning difficulties may encounter, and to provide some strategies that they can employ in planning mathematical experiences for their students, whether they are in mainstream classes or in special schools.

As students with mathematics learning difficulties may be learning their mathematics in many different settings it is important that the teacher initially identifies the point at which the individual students are operating. As we all know that the main goals of mathematics education is

- to develop a positive attitude towards mathematics and an appreciation of both its practical and its aesthetic aspects
- to develop problem-solving abilities and a facility for the application of mathematics to everyday life
- to enable the child to use mathematical language effectively and accurately
- to enable the child to acquire an understanding of mathematical concepts and processes to his/her appropriate level of development and ability
- to enable the child to acquire proficiency in fundamental mathematical skills and in recalling basic number facts.

The abstract and conceptual nature of mathematics poses particular challenges to students to learn mathematics effectively and thus mathematics stresses the importance of active learning, thus providing opportunities for students to manipulate, touch, and see objects as they develop their understanding of mathematical concepts. Learning within a group in which students are encouraged to talk about and explain how or why they did something will also support development of their own thinking about mathematics. An integrated approach to mathematics will help students to understand the relevance of mathematics in their daily lives.

It is important that the teacher is fully aware of the difficulties, both personal and academic, encountered by students in learning of mathematics.

Personal difficulties are very often underpinned by a poor self-image brought about by a long-term sense of failure. Failure may be one outcome of low intellectual ability, and can lead to slow progress. Slow progress is further aggravated by poor memory. Language and reading difficulties can confound students' difficulties with mathematics. Poor academic progress contributes to the poor self-image and lack of motivation, which in turn may lead to withdrawal or maladjustment of students. Low expectations can inhibit the student's effort and performance contributing to a cycle of failure. Teachers of students with failures in mathematics must strive to break that cycle by providing opportunities for students to experience success with mathematics.

Particular issues in mathematics difficulties for students

Many students in mathematics require a structured approach to mathematics. Opportunities to practice mathematics skills and concepts enable students to consolidate their learning. Direct teaching, using explicit strategies, is essential as some students may acquire inappropriate or incorrect strategies from incidental learning. While many students learn by working things out for themselves or observing how others work, when knowledge or skills are being used in a new context it is important to support students by making their learning explicit, since transfer of learning does not always take place automatically.

Students with a mild general learning disability are not

Students occasionally face difficulty in extracting the key features of a task and ignoring the less important ones. For example, they may focus on the numbers in a problem but not consider what they are being asked to do; hence they often just add all the numerals without considering the purpose. They may also focus on incidental information and fail to notice the salient feature of the topic. For example, they may be able to count by rote but fail to understand the use of numbers as labels for quantities *(I am 7, I live in number 7, she has 7 sweets).*

Early mathematical activities are aimed at encouraging more work in pre-mathematical activities to develop concepts before commencing formal number work. These activities are particularly beneficial for students with learning difficulties in mathematics. Teachers can frequently revisit these activities in age-appropriate settings.

The introduction of **number limits** encourages the consolidation of number facts and the development of the concept of place value. This is reflected in the reduction in the use of complex calculations and in the presentation of the same concept in different ways.

The emphasis on accurate use of **mathematical language** and understanding of symbols will contribute to a greater understanding of mathematics for all students. Students face difficulties in understanding the formal language of mathematics and fail to relate it with life problems. Learners should be given proper opportunities to consolidate their understanding of both symbols and mathematical language.

Although the calculators can be introduced to the class, but it is essential that students develop good **estimation skills** from the earliest stages if they are to use them efficiently.

The increased emphasis on the **use of manipulatives** (concrete materials) throughout the school, including the senior classes helps the students to understand the abstract ideas in different concepts of mathematics. Students can be thus provided support through the different materials to support their learning.

Students can also be given feedback immediately our through supplied answers for their immediate remediation of learning difficulties. Encouraging students to engage with open ended problems such as making scarves for teddy, building a house using a limited number of bricks, or working out how to spend a sum of money on food for a party will help them to realise that there are many ways to solve a problem, and that sometimes there is more than one 'right' answer.

The introduction of **'fun areas**' such as chance and mathematical trails allows for differentiated approaches, which can include all learners in exciting mathematical activities. On a trail some students may be looking for one digit numbers, while others may be seeking up to three digit numbers or adding numbers together.

The use of a **broad range of assessment tools** is essential in the accurate identification of students' strengths and needs. This is particularly true in mathematics where students may need to acquire certain skills or concepts before proceeding to more complex learning.

The emphasis on using a **variety of methods of recording students' progress** encourages differentiation of response, which recognises different learning styles. Some students may be able to give an answer verbally while others would benefit from producing a diagram or drawing. All students should have the opportunity to present their work in a variety of ways.

In the measures strand **answers should be verifiable** where possible to encourage understanding and the development of personal benchmarks. Handweighing and pouring activities using a variety of shapes of container will assist in this area. If students measure and label objects such as bookshelves, desks, etc in the classroom, they can build up visual benchmarks. For example,a student can stand beside the bookshelf and see that he/she is taller than a metre, or that the teacher is shorter than the door, which is more than two metres.

Mathematics means that students can come to see mathematics as relevant and connected to their own lives. Frequent reference by the teacher to mathematical elements in the course of learning is essential, for example measuring the distance from the classroom to the office using a trundle wheel in geography, using time words in history (before, long ago, a year), measuring jumps or distances run in Physical activities, etc.

1.5.1 Remedial strategies to deal with students learning difficulties in mathematics

The strategies outlined in this section use readily available concrete materials, allow students to experiment while learning new concepts, and encourage students to discuss their ideas on working out mathematical problems. In using the strategies that follow, the following approach should be taken:

(i) The teacher demonstrates the algorithm on the board while using the concrete materials, interacting with the students and posing questions as outlined in the individual strategies.

(ii) The students apply the algorithm using concrete materials until the steps are internalised and the concept is well grounded.

(iii) The students practice using the algorithm without concrete materials.

(iv) A student is guided to return to the use of concrete materials when problems occur.

The following list of general strategies may prove helpful in the teaching and learning of new facts, concepts, algorithms, and skills.

- Draw out important facts through classroom discussion.
- Try to plan for the consistent use of language and methodology throughout the school and, if possible, in the home.
- Encourage the student to work at a slow pace and in a very structured way. Avoid great leaps, taking very small, carefully thought out steps one at a time.
- Provide opportunities for the student to talk about a problem and to say how he/she can go about solving it.
- Encourage students to practice and apply the skill of estimation as often as possible.
- Provide the student with strategies for learning basic facts, and use a variety of table games and computer games to reinforce this knowledge.
- Use wall charts and display boards to show the facts and concepts that are relevant to the current classroom work.
- Provide regular opportunities for the student to use concrete materials until a new concept is well grounded.

- If a student has difficulties with a new concept and needs remediation, encourage him/her to return to using concrete materials.
- Choose figures that are easy to calculate when teaching a new concept or algorithm.
- Use materials that can be deconstructed where appropriate, for example using a bundle of ten lollipop sticks allows the students to see the ten single units, unlike a ten euro note. Initial use of lollipop sticks or Unifix cubes followed by the use of money means that the student is moving from a concrete to a more abstract use of materials.
- Use manageable materials where appropriate, for example money is far more manageable than lollipop sticks for larger numbers (hundreds), and cheques may be appropriate for even larger numbers (thousands).
- The students should only record the algorithm in the correct mathematical format when they are very familiar with the algorithm and even then only with easily managed numbers.

Math learning difficulties are common, significant, and worthy of serious instructional attention in both regular and special education classes. Students may respond to repeated failure with withdrawal of effort, lowered self-esteem, and avoidance behaviors. In addition, significant math deficits can have serious consequences on the management of everyday life as well as on job prospects and promotion.

Math learning problems range from mild to severe and manifest themselves in a variety of ways. Most common are difficulties with efficient recall of basic arithmetic facts and reliability in written computation. When these problems are accompanied by a strong conceptual grasp of mathematical and spatial relations, it is important not to bog the student down by focusing only on remediating computation. While important to work on, such efforts should not deny a full math education to otherwise capable students.

Language disabilities, even subtle ones, can interfere with math learning. In particular, many students have a tendency to avoid verbalizing in math activities, a tendency often exacerbated by the way math is typically taught in school. Developing their habits of verbalizing math examples and procedures can greatly help in removing obstacles to success in mainstream math settings.

Many children experience difficulty bridging informal math knowledge to formal school math. To build these connections takes time, experiences, and carefully guided instruction.

The use of structured, concrete materials is important to securing these links, not only in the early elementary grades, but also during concept development stages of higher-level math. Some students need particular emphasis on the translating between different written forms, different ways of reading these and various representations (with objects or drawings) of what they mean.

In sum, as a teacher, there is much we can and need to do in this area that calls for so much greater attention than we have typically provided.

1.6 Let Us Sum Up

The present unit discussed above presents the role of textbook in the context of teaching and learning in mathematics. The textbook is presented here as a major device of learning of mathematics to the students. It has been also highlighted how it is important for newly appointed teachers as well as catching upto the learning gaps of students. Further the role of other reference materials, workbooks, etc. has been also discussed .With the advent of 21st century the role of technology in teaching cannot be neglected. A proper description of teaching aids in mathematics has been presented here.

Lesson planning is a specialized task in teaching of any subject and it can determine the success and failure of any teaching learning scenario. The unit has tried to present a detail description of the basic of a lesson plan, the things taken care of and the guideline. The preplan strategies and postplan strategies have been also covered. A format lesson plan is also attached for better understanding. Mathematics is thought to be a difficult subject because of its enormous abstract concepts. Mathematics involves lots of abstract thinking, logical reasoning and inductive thinking. The lack of concrete materials makes it really a difficult subject to deal with both for the students and teacher. A section is thus devoted to identifying the mathematics difficulty in general and students' mathematics learning difficulties in particular. In the end some strategies has been suggested so as to face the learning difficulties of the student learners.

1.7 Check Your Progress

Now just try to go through the following questions and check your progress.

Q.1 Discuss the role and importance of classroom textbook in teaching and learning of mathematics ?

Q.2 What are the different audio-visual aids in mathematics? How does a mathematics teacher can use it for the effectiveness of his teaching?

Q.3 'Lesson planning determines the success and failure of teaching'. Explain?

Q.4 Write down the strategies and guidelines of lesson planning in mathematics?

Q.5 Prepare a lesson plan on any of the topic of secondary school mathematics?

Q.6 What are the different learning difficulties in mathematics?

Q.7 Suggest some remedial strategies for learning difficulties in mathematics?

1.8 Suggested Readings

NCERT;***Content-Cum-Methodology of Teaching Mathematics;*** NCERT, New Delhi (India).

NCERT; **National Focus Group on Teaching of Mathematics;** NCERT, New Delhi (India).

National Research Council ; ***Reshaping School Mathematics: A Philosophy and Framework for Curriculum*.** Washington, D.C.: National Academy Press, 1990

NCERT; **National Curriculum Framework;** NCERT, New Delhi (India).

Sidhu, K.S. **The teaching of Mathematics;** SterlingPub. New Delhi.

UNIT-V

Structure

1.0 Objectives

At the end of this unit, you will be able to:

- Explain the meaning and importance of evaluation in mathematics;
- infer the effect of evaluation on students;
- explain various levels of learning in mathematics;
- plan unit test in mathematics;
- Appreciate the usefulness of evaluation for learning and instruction;
- Describe different techniques of evaluation in mathematics;
- Understand the use of different other observational methods of evaluation;
- illustrate various types of questions used for evaluation;
- construct essay type and objective type test items in mathematics;
- illustrate the use of check lists and rating scales;
- Be able to understand the gifted students in mathematics and their educational needs;
- Acquire the idea of retarded students in mathematics and their educational needs; and
- Analyse any mathematics textbooks and judge is merits and demerits;

1.1 Introduction

The word "evaluation" is generally understood as some kind of assessment which means estimating the value of something. When the term evaluation is mentioned in the context of mathematics education, the first thought may perhaps be of tests and grades. But that is not all. Evaluation is much more than just tests which are only one of the many ways of assessing pupil performance. As a student, most of you would have perhaps thought that teachers used evaluation only to find out how their students had performed. Now, as a teacher of mathematics, you will understand how evaluation helps you to improve your own performance as a teacher as well.

This unit discusses the role of evaluation in the teaching and learning of mathematics. It also presents various techniques of evaluation with suitable illustrations. Further as we all know that all the students in the class are not with same mental ability. It is not only that we bother only for the backward students or retarded students but we should also should take care to identify the talented or gifted students of the class and nurture their mathematical abilities. This perspective has been also discussed here in brief. At last a detail idea has been presented on the analysis of a textbook in mathematics.

1.2 Evaluation of learning in Mathematics

Assessment and evaluation are integral component of any educational process. They not only provide feedback about learners, but also about the effectiveness of the curriculum, programmes and policies. We often use the term – Assessment and Evaluation interchangeably, but it is important for us to distinguish between them. The meanings and scope of these terms, as applied to educational setup, are explained below:

Assessment is defined as the process of obtaining and documenting the information about the subject, skills, attitudes and beliefs of the learners. It is an interactive process between students and teachers that informs teachers about the effectiveness of their teaching and level of students' understanding/ learning. Assessment provides feedback for the purpose of evaluation of learning outcomes and future performance. When we say that we are "assessing a student's competence," we mean that we are collecting information to help us decide the degree to which the student has achieved the learning targets. A large number of assessment techniques- formal and informal observations of students, paper-pencil tests, projects, assignments, etc. can be used to gather information.

Evaluation is defined as the process of making a 'value judgment' i.e., assessment about the worth of a student's performance. So through assessment of the student, the evaluation is carried out.

The distinction between assessment and evaluation can be understood by following example:

A teacher has to identify students writing ability, so that they can take part in a National level essay competition. In this case, the teacher has to make a value judgment about the writing skills of the students. For this the teacher has to 'assess' students' writing abilities by gathering information from the students' previously written essays etc. and compare with other students as well as with known quality standards of writing. Such assessment strategies provide information which can be used to judge the quality or worth of the students' writing.

The purpose of assessment is depicted in Fig. 1.

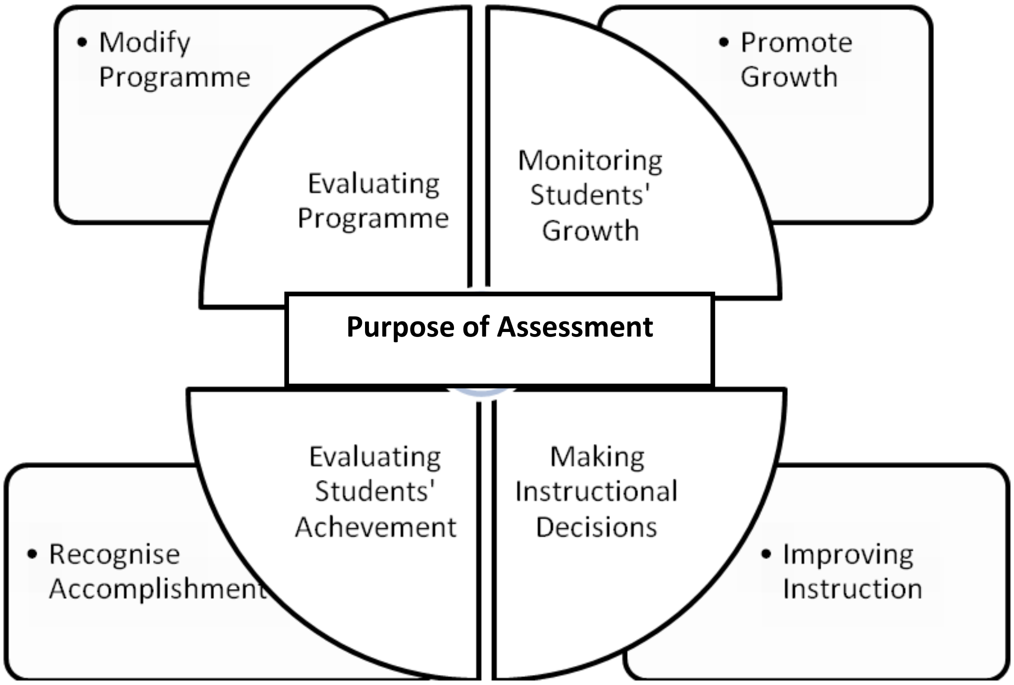


Fig. 1.

An assessment is authentic, is the assessment procedures match with what children are learning. It provides them with the feedback about their progress in mastering new knowledge. Authentic assessment acknowledges that learners learn differently and hence should get opportunity to express their learning in multiple ways.

1.2.1 Meaning of Evaluation

We, as teachers of mathematics, aim at making sure that our pupils learn mathematics and learn it well. The final test of a curriculum is its effectiveness in fostering learning. Every teacher has to find out the progress pupils have made towards accepted objectives.

How well have the pupils mastered the content and acquired necessary skills?

How well are the pupils able to explore and think and how well have they acquired the ability to solve problems?

Thus evaluation is concerned with the improvement of instruction. It involves decisions regarding the effectiveness of the total instructional programme.

What is the meaning of evaluation?

Your answers may include some or all of the following activities:

- giving tests,
- asking questions in the classroom,
- checking homework assignments, and
- organizing a quiz programme in mathematics.

All these activities are included in evaluation, but that is not all.

When we talk of evaluation in the mathematics classroom, we try to determine the amount and quality of pupil understanding and achievement in mathematics based upon clearly defined objectives. This means that a comprehensive range of objectives are evaluated rather than just the imbibing of the subject matter.

What constitutes the quality of mathematical learning?

A mathematics student is expected to learn facts, develop concepts, use symbols, master processes and procedures, learn to develop generalizations, apply mathematical ideas in real life situations, be able to reason deductively and so forth. It is likely that one student scores high marks through rote memorization while the other has acquired the ability to think and solve problems. Then who is a better learner? Obviously, the second one.

Thus, a wise teacher should evaluate the modes of learning employed by his pupils and not just what has been learnt. Modes of learning are as important as the content. High performance achieved through rote memorization is not preferred in evaluating the growth of the pupils. We use some procedures and techniques to collect data about pupil progress and growth to determine the extent to which these varied mathematical learning objectives have been achieved.

These procedures and techniques also form a part of evaluation. It will be useful to consider the distinction between evaluation and measurement. Measurement is the process of collecting data for the purpose of evaluation.

Now try to answer the question: "Why do we need to evaluate?" You may have thought of some or all of the following purposes or reasons of evaluation.

- to find out how much mathematics our students have learnt,
- to identify which students are weak in mathematics,
- to keep a record of their progress for reporting to the principal and to parents, and
- to recommend promotion to the next class or detention in the same class.

But this is not an exhaustive list. Evaluation not only says something about student performance, it reflects on the teaching also. Evaluation is important for both the teacher and the student. Let us find out how evaluation helps a teacher to teach better.

1.2.2 Evaluation for Improvement of Instruction

By now you know that before you teach you have to plan your lessons. While planning for instruction you have to keep in mind both the content as well as the students who have to learn. The students in your class are likely to have a wide range of previous knowledge and experience. They may also be operating at different levels of learning in the content you are planning to teach. For example, you want to teach applications of logarithms and one such application is simplification of exponential expressions of the type $(ab)^n$. You will have to assess whether your students can use a table of logarithms, recall laws of indices or solve a linear equation before you plan the learning experiences. Not only this, you will also have to assess the levels of understanding of your students. Levels of understanding are associated with structures of mathematical relationships.

For example, while solving $(8 \times 5\frac{1}{2})^{12}$ the student who chooses $(8 \times 11/_2)^{12} = (4 \times 11)^{12} = 44^{12}$ is better than the student who solves it as $(8)^{12} \times (5\frac{1}{2})^{12}$

The first student has higher level of insight into the relationship than the one who sticks to the original structure. This information about the individual student will help you design meaningful learning experiences for him. The teacher should be concerned about the identification of the levels of learning of his her students before teaching a new unit. This kind of evaluation is called **diagnostic evaluation.**

As far as planning the content is concerned, evaluation again can serve as a useful guide to you because for sequencing your content you need to know how long it takes to master a given concept, the relative ease or difficulty of tasks and the support material or teaching aids that are suitable for teaching a particular concept or topic.

During the period of instruction you need to monitor the learning progress of your students and diagnose their learning difficulties. Again you will be evaluating to get a systematic feedback about how your students are progressing with the lesson as well as about how your plans are working. This is called **formative evaluation.** You evaluate content a little more comprehensively by asking oral questions, giving classwork and using observation during the instructional phase.

After you finish teaching the unit you will be interested to determine the extent of your students' achievements and competence in the unit taught. In other words, you will evaluate their achievement. This is called **summative evaluation** and is done at the completion of a unit, term or year. It helps you to grade your students to provide data for school records as well as for reporting to parents. Again evaluation helps you to discharge your responsibility of reporting pupil progress.

Thus, you have seen how evaluation helps you to become effective at all the three stages of instruction. viz. planning, instructional and evaluative stages.

Let us try to see how evaluation helps our students.

1.2.3 Evaluation for Enhancement of Learning

For the student, an experience of evaluation is an exercise in learning also. For example, while a student is taking a test he has to think as well as perform operations. He/she usually experiences a feeling of concern and increased concentration. Therefore, his/her test responses are likely to be remembered longer than those given in a casual learning experience. In order to establish good records, students prepare well for their tests and besides reinforcing much of the earlier learning this exercise helps them in acquiring some new learning. A good revision for a test not only provides added practice but also a better understanding of the elements learned as well as their relationship with one another. Sometimes we evaluate students by informal methods like a quiz or an oral examination or interview. Such evaluation helps in determining a student's mental ability, his/her emotional maturity, his/her determination and background of experiences.

When students participate in scoring their own or their classmates' work, e.g., test paper, home work assignment or a scrapbook, they gain an additional learning experience. The process of discussing a class test when the test papers are finally returned gives the students an opportunity to discover their sources of error and then proceed to correct misunderstandings of facts, concepts, mathematical principles as well as errors in computations, etc. Evaluation thus has the potential to provide reteaching of weak links in the learning.

Students can also prepare items for tests or a quiz. Certainly, it will be an effective learning experience and the teacher can get information about the level of mastery of learning of individual students.

1.3 Techniques of Evaluation in Mathematics

An evaluation programme for a mathematics course or a unit has to be planned as an integral part of the curriculum during curriculum planning. It is only after the objectives of the course or the unit are stated in terms of student behaviour and appropriate learning experiences have been designed to bring about the desired changes in behaviour that decisions about evaluation can be taken. To assess the progress of students towards the accepted goals, a variety of evaluation techniques are applied. Broadly speaking, these are classified as testing and non-testing techniques.

The most common instruments that are used for evaluating student performance in a mathematics class are **tests.** The majority of instructional objectives pertain to the cognitive domain and can be appropriately evaluated with the help of direct test items. For example, tests can evaluate the knowledge of basic concepts, terms, processes and relationships in mathematics ,the application of mathematical knowledge and skills in problem-solving, skills of computation of using instruments, of drawing figures, graphs, etc., and analytical thinking as well. But there are other important aspects of mathematical learning which cannot be evaluated with the help of tests. We will talk about their assessment later in this unit. First let us know more about tests.

1.3.1 Tests

You have had the experience of being given tests by your teachers. Try to think of the purposes that tests can serve. You may be thinking of some or all of the following situations for which a test could be used :

- To predict a student's likely success in a mathematics course, we administer an aptitude test in mathematics.
- To assess readiness for learning a new unit, we administer pre-tests to measure the previous knowledge and experience of the students. The pre-tests forms a prerequisite for new learning. Such tests are also called inventories or surveys.

- Students use self-administering practice tests to check their own progress during an instructional period.
- We administer unit tests or achievement tests to assess the attainment of specific instructional objectives at the conclusion of a unit of instruction or a course.
- We administer diagnostic tests to identify the specific operational difficulties of a student who has persistent learning problems.

All these tests may be both teacher-made and published tests. Both are being used in schools and they have their own advantages and disadvantages. Published tests are prepared by experts, are usually easy to score and analyze and provide norms which help comparison of individual and group performance at state or national levels. Teacher-made tests have certain advantages over published tests. They are constructed keeping in mind the local context of teaching-learning situation. They can be constructed to keep pace with curriculum changes and are, therefore, likely to be more up-to-date than the published ones. They are comparatively inexpensive and provide a real learning experience to the teacher who constructs them. Such tests may also provide teachers with a basis far re-evaluating their instructional objectives. Such a process invariably helps them improve their instruction. Moreover, since teachers need to test quite frequently, they will have to construct their own tests.

The most commonly used teacher-made tests are achievement tests. These include unit tests, term tests and annual tests. Since you will be conducting a unit test most frequently, we describe the procedure for constructing a unit test in mathematics.

1.3.2 Unit tests

The purpose of a unit test, administered at the end of a unit of instruction, is mainly to measure the extent to which the intended learning outcomes of the unit have been achieved. These tests can also be used for providing feedback to pupils about their learning progress and assigning remedial work. They also provide feedback to the teacher about how well the instruction of the unit has gone. Thus unit tests serve the purposes of both formative, as well as summative evaluation. In a sequential subject like mathematics these tests serve the functions of pre-tests also; for example, a unit test on percentage can serve as a pre-test for the unit on profit and loss.

The construction of a unit test involves the following steps:

- Planning the test.

- Item writing for the test.
- Assembling, administering and scoring the test.

1.3.3 Planning a Unit test

While planning a unit test you will need to make decisions about the following:

- objectives of the unit,
- number of items in the test, and
- type of items to be constructed

A unit is usually a class test and the time available for such a test is limited to about 40 minutes. You may therefore cover only two or three important objectives in one unit test. Once you have decided which objectives are important for the content of the unit, list them. Thereafter, write out the behavioural specifications of these objectives in terms of student behaviour. You have learnt how to write behavioural specifications of instructional objectives in Unit 2 of this course. Once the objectives are specified in terms of student behaviour you have to decide how many questions or items you need to set for the test.

To ensure that the desired numbers of items are set for each objective, you must develop a table of specifications or the test blueprint. In this table, we usually write behavioural specifications horizontally to head the columns and content areas vertically to head rows. The specifications represent the number of items to be used for each objective. An example of a table of specifications is given below.

TABLE OF SPECIFICATION
UNIT : TRIGONOMETRY **CLASS : IX**

Behaviour/Content	recall	identify	change	find	verify	
Magnitude of an angle	4	4	-	-	-	8
Unit measure of an angle	2	-	4	-	-	6
Trigonometric ratios	4	-	2	-	4	10
Variations in trigonometric ratios for $0 \leq \theta \leq 90°$	2	2	-	-	-	4
Trigonometric ratios of specific angles	2	2	-	4	4	12
Angles of elevation and depression	2	-	-	2	-	4
Simple cases of heights and distances	-	-	-	4	2	6
	16	8	6	10	10	50

Here, the teacher has decided to set four items for recall of facts about the magnitude of an angle, two items for a comparison of the variations of trigonometric ratios as θ changes from 0 to 90°, four for verification of identities involving trigonometric ratios and so on. There are no set rules for determining the number of items that are enough to cover an objective but relatively more important objectives need to be covered by more items. In the illustrated table of specifications, more items, forty per cent, are set to cover the application objective because, for an elementary course on trigonometry, application is an important objective. The general rule is that the more the number of items the better the reliability of scores.

1.3.4 Constructing Test items

For constructing test items you will need to decide first what type of items to prepare. For this you will primarily be governed by the kind of student behaviour that is specified in the objectives. Let us turn to the aforementioned table of specifications. For demonstrating the achievement of the objectives of knowledge, understanding and application, the teacher expects a student to be able to do the following.:

- recall, e.g., values of trigonometric ratios of specific angles, fundamental identities like $\sin^2\theta + \cos^2\theta = l$, the definition of an angle of elevation, etc,
- identify, e.g., the position of a revolving line after tracing out a given angle, variations in the values of trigonometric ratios as θ changes from 0 to 90° and so on;
- change, e.g., angles from degrees to minutes to seconds and vice versa or one trigonometric ratio to another and the like;
- find, e.g., values of, expressions involving trigonometric ratios of 0°, 30°, 45°, 60°, 90°, heights and distances of inaccessible objects and so on; and
- verify, e.g., trigonometric identities, etc.

To assess the achievement of these objectives we can use either essay type items or objective type items or both. In a mathematics test, we use essay items only when we want to assess communication skills besides logical ability and-precision in thinking. In these items students are required to select, organize and integrate information before writing out the answers as you can see in the following examples of essay items.

Item 1 : From an external point construct a tangent to a given circle.

This item requires the student to draw the construction, discuss the steps of construction and write out the proof.

Item 2 : Prove that $\sqrt{3}$ is an irrational number.

In this item, the student is expected to show an understanding of deductive proof as well as demonstrate the skill to communicate each step logically.

Item 3 : A boat is being rowed away from a cliff 150 metres high. From the top of the cliff the angle of depression of the boat changes from 60° to 45° in two minutes. Find the speed of the boat.

This item requires the student to translate the given information correctly into symbolic form and then write out the solution giving the rationale for each step.

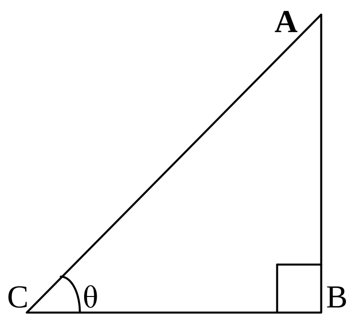
Essay items are no doubt valuable exercises for students but they consume a lot of testing time. For a unit test, therefore, we may have very few of these items if it is necessary to have them at all. Moreover, scoring of these items objectively is also difficult because there are wide variations in answers as far as accuracy and completeness are concerned.

Objective type items: Objective type items are used when we want to test students' knowledge and understanding of facts and relationships in mathematics. These include completion, true/ false, multiple choice and matching items. Completion items are suitable for testing the recall of terms, 'facts and relationships as well as for computational skills. The student has to write out the answer but it has to be very short and can be a word, symbol, number or phrase. Its scoring can be fairly objective. Objective type test items are useful and convenient for unit tests.

Examples of completion items:

Write the answers to the following questions 1 to 10 in the blank space provided on the right hand side of each question.

Answer questions 1 to 5 in respect of ΔABC given below :



1. The value of cos A = ...
2. The value of tan B = ...
3. The value of cosec A = ...
4. The value of tan **A** = ...
5. The value of sin B = ...
6. If θ is acute and tan θ = 5/12 then sin θ + cos θ = ...

7. If θ lies between 0° and 90°, sin θ lies between ....
8. sec 45° + cosec 45° = ...
9. $1 + \tan^2 \theta$ = ...
10. The maximum value of $2 \sin \theta + 3 \cos \theta$ = ...

In all these items the focus is on the answer and if we want to assess the method of solution as well as the answer we use short answer items. These items can be solved in limited steps, usually three to five steps, and with a properly defined scoring procedure they can be scored quite objectively.

Examples of short answer items

Solve the following questions using the minimum number of steps.

I. If x = 30° and y = 60°, verify that

c o s (x + y)= cosxcosy-sinxsiny

2. If sin θ = p/q find cos θ in the simplest form. I

3. If sin θ - cos θ = 1, find the maximum value of sin θ + cos θ.

4. A straight iron rod leaning against a vertical wall makes an angle of 60° with the ground at a distance of 5 m. from the wall. Find the length of the rod.

5. If tan θ = k/l determine the value of

$$\frac{a \cos \theta - b \sin \theta}{a \cos \theta + b \sin \theta}$$

True/false items or alternate response items are based on statements which are either clearly true or false. Students are required to select the response from true or false, right or wrong, yes or no. These items provide students opportunities for guessing the right answer. They can, however, be scored quickly and objectively. Since they normally take little time to answer they can be used for testing a large amount of subject matter in a relatively shorter testing time.

Examples of true/false items

In the following statements 1 to 7 some are true and some false. Write 'T' against each true statement and 'F' against each false statement in the blank space provided for the answer.

1. Counter-clockwise rotation of a revolving line yields a negative angle .....
2. As θ increases from 0° to *90°*, the value of cos θ decreases from 1 to zero…..
3. If tan x = I , then x is equal to *90°* ....
4. sin A + cos B = sin (A + B) ....
5. If tan θ + cot θ = 2 then tan2 θ + cot2 θ = 2 ....

6. If the length of the shadow of a pole is equal to its height then the angle of elevation of the sun is 90° ....

7. The angle of elevation is numerically equal to the angle of depression ....

Multiple choice items require a student to select an answer from three, four or five given response options out of wh.ich only one is the correct answer and the rest are distractors. This type of item can be used for a wide variety of objectives and can be scored quickly and objectively.

Sample multiple choice items

In each of the questions I to 6 there are four possible answers marked at A, B, C and D. Only one of these answers is correct. Write the letter given against the correct answer in the bracket on the right hand side of each question.

1 **.** A revolving line starts from OX and traces an angle of 520°. In which quadrant will it lie?

(A) First, (B) Second, (C) Third, (D) Fourth ( )

2. What is $1 - \sin^2 30°$ equal to?

(A) $\cos^2 30°$, (B) sin 60° (C) --cos2 60°, (D) None of these ( )

3. When $0° < \theta < 90°$ the maximum value of- $\sin\theta + \cos\theta$ is

(A)$1/\sqrt{2}$, (B) 1, (C) $\sqrt{2}$ (D) 2 ( )

4. Which one of following is possible?

(A)$\sin\theta = 2\sqrt{2}$, (B)$\cos\theta =-2$, (C) $\sec\theta =20$, (D)$\operatorname{cosec}\theta = 1/20$ ( )

5. If $\tan\theta = 3/4$, then which one of the following is true ( )

(A) $\sin\theta = 3$, $\cos\theta = 4$

(B) $\sin\theta$ and $\cos\theta$ can have many values

(C) $\sin\theta = 3/7$ and $\cos\theta = 4/7$

(D) The values of $\sin\theta$ and $\cos\theta$ cannot be found out from the given information.

6. A flagstaff stands on a horizontal plane and from a point on the ground at a distance of 90 metres its angle of elevation is 45°. The height of the flagstaff is ( )

(A) 45m, (B) $90/\sqrt{2}$, (C) $90\sqrt{2}$m, (D) 90 m.

Matching type items, like multiple choice items, require a student to select an answer from a set of given options. To make these items effective try to make the responses as homogeneous as possible.

Sample matching items

Watch the trigonometric ratios of column I which are equal to those in column II like N – X where N = 1,2,3,4, X = A,B,C,D,E, and write the answers in the space provided on the right hand side.

Col. I	Col. II
1. tan 45°	A. cos 60°
2. sin 30°	B. sec 45°
3. tan 60°	C. cos 0°
4. cosec 45°	D. cot 30°
	E. sin 90°

In column I are given statements which are true for the ranges of values of θ given in column II. Match them like N-X, where N = 1,2,3,4, and X = A,B,C,D,E,F.

Col. I	Col. II
1. $\tan\theta \geq 1$	A. $0° < \theta < 90°$
2. $0 < \sin\theta < \sqrt{2}/2$	B. $0° \leq \theta < 90°$
3. $0 < \cos\theta \leq 1$	C. $0° < \theta < 45°$
4. $\tan\theta > 0$	D. $0° < \theta \leq 90°$
	E. $\theta \geq 90°$
	F. $45° \leq \theta < 90°$

After having decided the number and type of items to be included in the unit test, you have to write the test items. These include some basic rules like avoiding the use of long and ambiguous statements, extraneous clues that lead to the answer, negative statements, etc.

1.3.5 Assembling, Administrating and scoring the test

Dents find easy to understand how the questions are to be answered and where they have to record their answers. You should also be able to locate the answers and score them conveniently. For this you may find the following guidelines useful:

- Group the same type of items together, i.e., all multiple choice items at one place; all true/ false items together and do the same with short answer and matching items.
- Arrange each set of items of one type from easy to difficult, that is, among the multiple choice items the easiest item should be the first and most difficult the last. Do the same with all other types of items. ,
- Number all items starting from the first, i.e., 1,2,3, ..., etc., to the last item.
- Write all items legibly and avoid splitting an item on two pages, i.e., writing a part of an item in the bottom line of the page and the other part on the next page should not be done.

- Before each group of items, write simple and clear instructions for students telling them how and where to write the answer and read directions in sample items given earlier. Each set of items needs different directions.

Administration : It is better if you give each student a copy of the test to work on rather than make them/ copy the test written on the blackboard. Make sure that all the students are seated in a manner that they have reasonable elbow room and desk space and also ensure sufficient light and ventilation in the room where the test is conducted.

Scoring the test: Prepare an answer key for all the objective type items of the test in advance. For short answer items write out the solution steps and marking procedure for each step. For essay items also prepare model answers in advance and use the same uniformity in scoring each paper.

After the answer sheets are scored it is desirable to tabulate and analyse the scores to get information about the level of performance of the class as a whole as well as of individual students. The analysis of test results can also provide you information about the individual items, viz., their difficulty value and discrimination index, as well as the total test, viz., reliability of the test, etc.

Now let us talk about other techniques of evaluating mathematical learning besides tests.

1.4 Use of Observational and other methods of evaluation

There are some important aspects of mathematical learning which cannot be evaluated with the help of tests. Try to think of such aspects and list them in your note book. You are right if you have listed the following:

- Interest in mathematical activities and computations and in reading mathematical material other than the prescribed ones.
- Curiosity about new mathematical ideas, short cuts and alternative methods of solution.
- Work habits of neatness, accuracy and speed in doing computations or drawing figures, graphs, etc.
- Appreciation of the contribution of mathematics to daily life and to civilization.

Evaluation of these behaviours is not easy because they cannot be measured by direct questions. They can, however, be assessed indirectly by using observational methods carried out both inside as well as outside the class. Mathematics teachers do observe their students at work and gain impressions like "student x likes geometry more than algebra', "student y is quite an active participant in the mathematics club", "student z works very neatly and systematically" and so forth. Such chance observations, though carried out in natural situations, suffers on two counts: one, that we remember only a part of what we happen to see by chance and, two, our own attitudes tend to influence such chance observations making them biased. You can, however, correct this bias if you do the following:

- Plan what student behaviours are to be observed and make a list of them well in advance.
- Concentrate only on one or two behaviours.
- Carefully record and summarize the observation soon after it is made.
- Observe students at scheduled times.

The observational methods that you will find useful are rating scales and check lists. We shall only illustrate their use here.

Rating scales: A rating scale consists of a set of behavioural attributes that are to be assessed and some kind of a scale indicating the degree to which each attribute is present. For example, if you want to assess the extent to which a pupil takes interest in mathematical activities, then you have to do the following :

- Write out the specific behaviours that a student should demonstrate so as to indicate his interest in mathematical activities.
- Frame items based on these behaviours for the rating scale. *** .**
- Indicate the rating categories and scale.

You can have a three-category or a five-category scale or more categories if you want finer distinctions. A five-category rating for the above mentioned scale would be, for example, never, seldom, sometimes, usually, always, where never is 1, seldom is 2, sometimes is 3, usually is 4 and always is 5. Indicate the strengths of these categories

SAMPLE RATING SCALE

Directions : Indicate the degree to which the pupil takes interest in mathematical activities by encircling one option on the right side of each item.

1. Contributes articles, games, puzzles,etc., for the wall magazine or school journal	Never	Seldom	Sometimes	Always
2. Participates in activities of mathematics club	Never	Seldom	Sometimes	Always
3. Uses short cuts for solving problems	Never	Seldom	Sometimes	Always
4. Seeks clarification of doubts in problems attempted from sources outside the prescribed books	Never	Seldom	Sometimes	Always

5. Brings applications of mathematics to class	Never	Seldom	Sometimes	Always
6. Solves mathematical puzzles	Never	Seldom	Sometimes	Always

Check lists: A check list is a simple method of recording whether a particular characteristic is present or absent. It provides only two categories, yes or no. We can use check lists to assess procedures as well as products, e.g., to assess whether a pupil is able to use geometrical instruments effectively as well as to assess whether the figure drawn is neat and accurate. Suppose we want to assess if pupils can use geometrical instruments effectively. Here also we first specify what should the student do to demonstrate effective use of geometrical instruments. List the behaviours as : selection of appropriate instruments, quick and correct use of instruments, measuring accurately, drawing neatly, putting back instruments in the geometry box.

SAMPLE CHECK LIST

Directions : Circle Yes or No to indicate whether the skill of using geometrical instruments has been demonstrated.

1. Selects appropriate instruments.	Yes	No
2. Uses them correctly and with speed	Yes	No
3. Measures accurately	Yes	No
4. Draws neatly	Yes	No
5. Puts instruments back in the geometry box	Yes	No

We can observe the pupils while they are doing the class assignment "construct a tangent to a given circle from a point outside it" and record entries on the check list.

Also, you can involve your students in the process of developing check lists or rating scales. A discussion with the students about what, after all, indicates interest in mathematical activities or effective use of geometrical instruments will cause them to think more of the standards they have to strive for. Their involvement in setting goals can motivate them to work harder towards the achievement of these goals.

Portfolios

Portfolio is a collection of evidence of a person’s skills. For the purpose of assessment, a portfolio is a limited collection of students’ works which gives evidence of learner’s meaningful learning and demonstrates their growth over span of time.

Advantages of Portfolio

- Promotes problem-solving and thematic approach.
- Shows growth in acquiring knowledge and skills.
- Encourages students to collect, organize and reflect on their own learning.
- Provides scope for multiple means of knowledge representation.
- Encourages performance in non- traditional and non-language dependent medium.

For these reasons, portfolio are increasingly becoming popular as an authentic means of assessment.

Content of mathematics portfolio

A variety of items can be included in a portfolio which allows the students to work over a span of time. Some of the suggested items are :Open ended question, A report of group project, problems posed by the students, a small project, a book review, excerpts from students journal, newspapers and magazines articles, a mathematical research, problem-solving tasks, self evaluation and peer evaluation , etc.

Projects

A project is a motivated problem, solution, of which requires thought and collection of data and its completion results in the production of something of value to the students.

Project enables learners to conduct real inquiry in an interdisciplinary manner. It promotes problem-solving in mathematics and connects it to real life applications,

Projects in mathematics provide opportunity to observe, collect data, analyze, organize and interpret data and draw generalization.

A project could be individual or group project and could be presented in the form of a document, report and/or a multimedia presentation.

Rubrics

A ‘rubric’ provides written guidelines by which student’s work is assessed. The term rubric is defined as “a set of guidelines for assessment which states the characteristics and /or dimensions being assessed with clear performance criteria and a rating scale”.

A scoring rubric consists of a fixed scale, a list of characteristics describing performance for each of the points on the scale, a clear performance target for the students. Rubrics are useful for both learners and teachers as they make evaluation less subjective. Example of mathematics rubrics for problem-solving task

Level 4	Level 3	Level 2	Level 1

• **Shows complete understanding of mathematical problems, concepts and principles.** • **Uses appropriate mathematical terminology and notations** • **Executes algorithms completely and correctly**	• Shows nearly complete understanding of mathematical problems, concepts and principles. • Uses nearly correct mathematical terminology and notations • Executes correct algorithms but computation as minor errors	• Shows partially understanding of mathematical problems, concepts and principles. • Uses incorrect notations • Executes incorrect algorithms	• Shows no understanding of mathematical problems, concepts and principles. Needs revisiting the concepts • Does not use any notations • No algorithm is executed

Anecdotal record

Anecdotal records are written records/description about the learner maintained by the teacher which generally includes observations and narratives. They not only help in identifying strengths and weakness of the learners, but also his progress over a period of time.

Journal Writing

Journal writing helps the learners to reflect on their own learning in their classroom and provide opportunity to express their opinions. This process of writing about learning encourages learners to analyze what they have learnt in the classroom. They provide teachers with insights into the ways in which students have constructed knowledge and into their levels of conceptual understanding.

1.5 Education of gifted and the retarded learners in mathematics

Mathematics itself by its nature is thought to be a difficult subject to be studied. Regardless of this there are students who perform excellently well in mathematics. The nomenclature thus used for them is that they are gifted learners. Gifted learners have got immense potential to excel in any particular or in general area of study. The system in our school setup hardly bothers to cater the demands of these categories of learners .As there is lack of sufficient attitude deficiency for nurturing the potential of individual learners it becomes very inefficient practice for handling the giftedness of the individual. There are also cases where we find that some or more students lag behind the normal speed of learning of mathematics they are treated as retarded or backward children. The task of identifying and helping retarded children is more important than to handling of giftedness. Inappropriate teaching mechanisms, biasedness to a particular group of students, system structure never give the scope to help and guide these backward students in mathematics. The following section tries to identify to discus on these particular issues.

1.5.1 Education of the gifted students

There is a large group of students who falls in the group of gifted students in mathematics. Their performance in mathematics and/sometime other areas too are above excellence. The need and demands of these groups of students are much more. It is thus required to carefully identify these students in a given setup and nurture their talent and potential to the fullest.

Characteristics to look for when identifying Mathematically Gifted Students

There are many characteristics to consider when identifying which students are mathematically gifted. The following descriptors of characteristics of highly able mathematics students should be viewed as examples of indicators of potential. Few students will exhibit all characteristics and these characteristics can emerge at different times as the child develops cognitively, socio-emotionally, and physically.

The highly able mathematics student should *independently* demonstrate the ability to:

- display mathematical thinking and have a keen awareness for quantitative information in the world around them.
- think logically and symbolically about quantitative, spatial, and abstract relationships.
- perceive, visualize, and generalize numeric and non-numeric patterns and relationships.
- reason analytically, deductively, and inductively.
- reverse reasoning processes and switch methods in a flexible yet systematic manner.
- work, communicate, and justify mathematical concepts in creative and intuitive ways, both verbally and in writing.
- transfer learning to novel situations.
- formulate probing mathematical questions that extend or apply concepts.
- persist in their search for solutions to complex, "messy," or "ill-defined" tasks.
- organize information and data in a variety of ways and to disregard irrelevant data.

- grasp mathematical concepts and strategies quickly, with good retention, and to relate mathematical concepts within and across content areas and real-life situations.
- solve problems with multiple and/or alternative solutions.
- use mathematics with self-assurance.
- take risks with mathematical concepts and strategies.
- apply a more extensive and in-depth knowledge of a variety of major mathematical topics.
- apply estimation and mental computation strategies.

It is important to realize that these variables are not fixed and need to be continually developed. Unfortunately, there is no single method for identifying gifted and talented students nor for assessing their performance. Ways of identifying mathematically promising students include:

Observation—while the students are working, particularly in problem solving situations of increasing difficulty or those designed to elicit the characteristics listed above.

Portfolios—students need access to exemplars from other students and the scoring rubric should include:

- patterns noted and generalized
- predictions made and verified
- interesting related problems posed and investigated
- measures of creativity—
    - fluency (number of different solutions)
    - flexibility (variety of solutions)
    - originality (uniqueness of solutions)
    - elegance (clarity of expression)

Questioning—individually, in small group, or whole class:

- Student interview
- Parent information
- Student interest/peer survey
- On-going assessment

- PADI diagnostic instruments, such as Rating Student Potential Teacher Checklist
- Diagnostic Thinking Tasks
- Math logs or journals

How to Teach Mathematically Gifted Students

When planning instruction for gifted and mathematically promising students, there are questions that need to be asked:

- Do the opportunities provide for the wide range of abilities, beliefs, motivation, and experiences of students who have mathematical promise regardless of their socioeconomic and ethnic backgrounds, and do the opportunities meet their continuum of needs?
- Are curriculum, instruction, and assessment qualitatively different and designed to meet the differing needs of promising students?
- Are there resources, projects, problems, and means of assessment that allow for differences in the level of depth of understanding and engagement?
- Are there appropriate opportunities in mathematics that have clearly defined, comprehensive, integrated goals—that are not simply isolated activities?
- Are the opportunities available to all interested students and in all schools?

“Children with special abilities and talents are part of the human mosaic in our schools and communities. They typically learn at a pace and depth that set them apart from the majority of their same-age peers. Because they have the potential to perform at high levels of accomplishment and have unique affective and learning style need when compared to others of their age, they require instructional and curricular adjustments that can create a better match between their identified needs and the educational services they typically receive."

Sections excerpted from Developing Mathematically Promising Students, edited by Linda Jensen Sheffield, National Council of Teachers of Mathematics, Reston, Virginia.

1.5.2 Education of the retarded students

As there are a prominent group of learners who excels brilliantly in mathematics there is another a large mass of students whose performance is very poor in mathematics these categories of students are often termed as retarded students, backward students or slow learners in mathematics. Our method of teaching of a group or a class seldom gives scope for individualised attention to these learners and te problems persists with these group of learners and even exgragated in the future. Thus it is required to identify these learners and provide them required support to be at par the normal group of learners.

Characteristics of retarded learners:

1. Functions at ability but significantly below grade level.
2. Is prone to immature interpersonal relationships.
3. Has difficulty following multi-step directions.
4. Lives in the present and does not have long range goals.
5. Has few internal strategies (i.e. Organizational skills, difficulty transferring, and generalizing information.)
6. Scores consistently low on achievement tests.
7. Works well with "hands-on" material (i.e. Labs, manipulative, activities.)
8. Has a poor self-image.
9. Works on all tasks slowly.
10. Masters skills slowly; some skills may not be mastered at all.

Causes and remedial strategies for retarded students

1. Physical causes

There may be some physical cause, such as poor eye-sight, defect in the hearing organ, stomach trouble, headache or any others physical ailment, which does not allow the child to concentrate on his studies. Mathematics needs special concentration. The remedy of almost all these causes lies with the doctor or physician, but some sort of physical exercise may also help the child. The teacher has to refer the child to the doctor, some other medical expert or the PTI according to the nature of the case. He has to try to make available all the possible remedies. If it is a case of serious physical deficiency the teacher should try to persuade the parents to get their ward admitted to a special institution for the physically handicapped children of that type.

2. Mental causes

The backwardness may be due to some mental causes. These causes may be almost inborn and/or environmental. The child may have low I.Q.,some mental ailment, mental dissatisfaction, domestic problem, mental conflict, sense of insecurity, inferiority complex, lack of interest in mathematics or a dominant interest in something else. A case of some simple mental problem can be tackled with some chances of success by the teacher. But complicated cases will have to be passed on to a psychologist or a psychoanalyst for thorough diagnosis and treatment. An attitude of love and sympathy and preparedness to help the children on the part of the teacher will always be helpful in the treatment of such cases.

3. Distaste for the subject

A distaste of the subject can be another cause. This distaste may either be natural or acquired. If it is inborn, the teacher's may be efforts may go waste. But if it is an acquired one, it may be mostly the teacher's fault. If he fails to develop a feeling of attachment between the child and himself, the only outcome is the child's distaste for the subject. Heaviness of the syllabus, toughness of the subject and difficulty of its problems tend to produce this distaste. A genuine taste for the subject will be developed through the teacher's patience and persistence. He should never be in a hurry to pronounce a case as hopeless and backward. He should never allow the idea 'once backward always backward' to enter his mind'

4. Doubts about fundamentals

Sometimes the child develops some doubts about the fundamental of the subject, and these doubts hinder his progress throughout. The teacher should be very careful while teaching th ese fundamentals. He should not hesitate to explain them over and over again if required. Only clear understandings of these fundamentals can build a sound foundation for mathematical learning.

5. Wrong influence of home

Influence of the home may also create distaste and backwardness. Some parents unintentionally provide negative suggestion to their children. Some of them are in the habit of saying that they never liked mathematics or they never wanted to study it, or were never be able to pass in it or that failure in mathematics has been the tradition in their family. Even the educated parents commit the same mistakes. These pronouncements are likely to have an adverse effect on the child's attainment in the subject. Parents must be made conscious of this adverse effect and their duty in this manner.

6. Teacher's behavior

The teacher's unbalanced behavior may also be one of the causes. If he is very lenient, some of the clever and mischievous students may get undue advantage of it and may become backward due to continuous inattention and non- seriousness. If he is very strict and gives heavy punishments unsparingly, some of the feeble minded students may get disheartened and discouraged. They may start disliking the teacher and consequently the subject. The teacher should never forget that is behavior is very important.

7. Mid session changes

The change of school or even the change of teacher may not suit some of the students. Whenever the change of school is unavoidable, the parents and the teachers have to remain on guard till the child's complete adjustment with the change. Changing the teacher in the middle of a session should be avoided as far as possible.

8. Teacher's reputation

The subject teacher's bad reputation in the school and neighborhood may also affect some of the students adversely. The teacher must earn respect and reputation. He should be a source of inspiration for his students ijn every respect. He should carefully maintain his scholarship and character.

9. Apathy towards method

Apart of the student's apathy towards the teacher's most favorite method of teaching may result in their retardness. The teacher should not always stick to the same method. Students like 'newness and novelty'. The teacher should always be prepared to adjust his method to the learner's like. He may have to devote extra time and attention to the student who is lagging behind. Slow learner deserves the teacher's individual attention and help. The teacher should so conduct himself that the slow learners never feel that they are being disowned or being left behind.

10. Defective handwriting

Defective hand at writing and geometrical constructions may be the cause of unsatisfactory performance in certain cases. Their weakness does not therefore pertain to mathematics. The cooperation of language teacher may be sought in their case. The teacher should make it a point to insist on neatness of hand and constructions in such cases.

11. Lack of practice

Of these students, some may need more practice and drill than others. The teacher should not overlook their need. The intelligent one may pick up an idea after solving only one problem, whereas the slow learner may need solving two or three problems to grasp it. The teacher has to adopt a via-media.

12. Neglect of home work

When the child does not get sufficient time at his home for home work, he is likely to lag behind. Regularity in home work should be ensured. The parents should provide full cooperation to the teacher in this regard.

13. Irregular attendance

If the child remains unavoidable irregular or absent for a long time, there is chances of his failing a prey of retardness. The facility of extra-coaching for sometime should be provided to such students so that they can fill up their gaps and catch up with the other class-fellows. Such temporarily retarded students should not be left to themselves. The provision of extra-classes made in most of the schools near the examination, enables many backward students to get through.

14. Examination system

Faulty examination system may also cause ignore this subject se non-seriousness and retardness among some students. If passing in mathematics is not compulsory at the time of passing the final examination, some students may ignore this subject. Therefore it should be treated as a compulsory in this respect.

1.6 Analysis of textbooks in mathematics

Unfortunately, no textbook is perfect! Effective teachers carefully analyze their textbook and are aware of its strengths and problematic areas. Once teachers think about the text, they can supplement material where needed, better direct instructional strategies while using the text, anticipate difficult sections for students and plan accordingly.

Considerations for Analyzing and Evaluating Content-Area Textbooks

A. PHYSICAL LAYOUT

Do the authors organize the material in a clear and meaningful manner? Give specific textual evidence to substantiate your description of the book's overall layout.

- Are the chapters and subsections well organized?
- Does the table of contents represent a logical development of the subject matter?
- Are there common organizational features among all of the chapters that help students organize new information they learn? (e.g., headings/subheadings, graphic organizers, tables, proposed objectives, etc).
- Do the pictures, graphs, and charts work well with and support/extend the text itself? Do they vividly illustrate the concepts covered?
- Are the captions present and helpful?
- Is the size of the print appropriate?
- Are there any "special features" at the beginning and end of the book that make the text especially useful?

B. CONTENT

Do the authors present the material in an accurate, meaningful, and engaging manner? Give specific textual evidence to substantiate your description of the content.

- Do you like the questions asked of students before, during, and after the chapters? Do they call for mere literal recall, or do they stimulate students to think critically, apply concepts, draw connections across ideas, or apply to one's own interests and experiences?

- Are the examples ones that students can relate to?
- Does the text include quotations from primary sources and authorities to support and add interest?
- Do the illustrations and examples fairly represent race, ethnicity, gender, and class? Are the representations of people non-stereotypical?
- Are multiple and diverse perspectives offered in relation to the content?
- Do the chapters contain opportunities for self-assessment? Are there multiple formats for self-assessment?

C. CHAPTER SUMMARIES AND SUPPLEMENTARY MATERIALS

Do the authors provide explanations of the intended goals and overall purpose for using the material? Give specific textual evidence to substantiate your thinking.

- Is each chapter's main idea(s) or purpose for reading explicitly stated at the beginning?
- Do the chapters contain study guides, summaries, or other special features to help review the major concepts during and/or after reading?
- Are there supplemental materials, such as workbook exercises, CD-ROM activities, companion web resources, etc? If so, do they require students to do things that will actually further their practice of a skill or their knowledge of the subject? Do you consider them a valuable use of your and your students' time or just busy work?
- Are suggestions and resources given for a teacher's further exploration using primary and secondary sources? (e.g., annotated bibliography, extension activities, etc.)

D. VOCABULARY

Are technical terms highlighted, defined well, and explained adequately for adolescent readers when they are first introduced? Give specific textual evidence to substantiate your thinking.

- Is there an appropriate number of new terms introduced in each chapter?
- Are there relevant application examples of new terms?
- Does the text provide necessary background knowledge by reviewing or reminding readers of previously acquired knowledge or concepts?
- Calculate the readability of the text. Is it appropriate for the grade level for which it is intended?

E. ADRESSING SPECIAL LEARNING NEEDS

Do the authors suggest other resources and activities for students motivated to explore the area or for students who have difficulties with specific objectives or specific tasks?

- Are the exercises appropriate for a range of populations or might some of the assignments need to be adapted to support different kinds of learners?
- Is the material expressed in multiple ways (e.g., words, illustrations, photos) to support different kinds of learners?

F. POTENTIAL PROBLEMS OR CONCERNS

What potential problems or concerns might you have for using this textbook and what are some suggestions for remedying potential problems?

- For students (language, content, assessments, etc.)
- For teachers (too much material to be covered/too little information/too complex / too superficial a treatment of the concepts, etc.

G. OVERALL EVALUATION

- Make a listing of the overall strengths and problematic areas in the text as a whole
- Would you recommend this text for usage in your school? Why or why not?

Some general mathematics textbook evaluation guidelines

1. Make sure the basic mathematic concepts are is covered. Look at the outlines for Algebra, Geometry, Algebra II with Trig, and Precalculus below as a guide. The wording and chapter arrangement may be different, but you should see these key ideas in the contents.
2. Is there a well-written table of contents? When doing math you need to be able to look up a concept.
3. Since you will be using the math textbook for homeschooling, look for math books with lots of worked examples. Go through some yourself and determine if they are easy to read and follow.
4. Look for plenty of problems to work. While your students does not need to work every single math problem in a textbook, having a lot to choose from makes the teachers job easier.
5. Does the math textbook have answers? We like the textbooks that have the answers to the odd numbered problems for the students and then have the solutions to all the problems in the teacher's manual. If they do not have the answers to many problems, you students will never know if they are doing the math correctly.

6. Look for some real world examples and problems. Do the problems tell about real situations? This real world approach is key to helping your students see how math connects to the real world.
7. Look at some chapters. Are the key points of the chapter outlined in boxes or color so that they stand out? This makes it easy to use as a math reference now and later.
8. Is there a teacher's reference that tells you how to use the math book for a course? If so, is it useful to you? Does it make sense to you?
9. Is there an index?
10. What does your student think? If you can, let them compare a few and ask which they like better.

Math materials (such as textbooks, videos, or computer based teaching) should all cover about the same material per course title. In fact, many textbooks will have the exact same chapter titles.

So you can do a pretty good job evaluating the coverage of material based on the chapter and subchapter titles. Here are some typical titles or subtitles that should be keywords in your comparison of material. Note that these topics may not match your math textbook exactly or be in the same order as listed below. But, a majority of the key mathematical points listed should be found. To use this guide go to the table of contents in your math book and look for these keywords. You should not have to search the entire text or videos to find them.

Examples:

Algebra Textbook Evaluations

Look for the following material or heading in the contents of your Algebra textbook.

- Variables Exponents
- Order of Operations
- Equations and inequalities
- Word problems (converting words into symbols)
- Real Numbers
- Adding
- Subtracting
- Multiplication
- Division
- Distributive property
- Linear Equations (might be equations in one variable)
- Graphing Slope Intercepts
- Point-slope and or slope-intercept formulas
- Systems of linear equations (or simultaneous equations)
- Linear inequalities
- Solving
- Graphing
- Absolute Values
- Exponents

- Products
- Divisions
- Scientific notation
- Polynomials
- Quadratic equations
- Factoring Radicals

Notes for use: All the material on the above list needs to be covered in algebra. Instead of moving fast, the students should understand these math concepts well.

Geometry Textbook Evaluations
Look for the following material or heading in the contents.

- Reasoning and Proof
  - Proofs
  - Deductive reasoning
  - Direct and indirect proofs
- Lines
  - Parallel and Perpendicular Lines
  - Angles
- Triangles
  - Congruent
  - Isosceles
  - Equilateral
  - ASA and SAS
- Quadrilaterals
  - Parallelograms
  - Rectangles
  - Squares
  - Trapezoids
- Area
  - Squares
  - Rectangles
  - Triangles
- Similarity
  - Ratio and proportion
  - Similar figures
- Right Triangle Trigonometry
  - Pythagorean Theorem
  - Proportions
  - Tangent, Sine and Cosine
- Surface Area and Volume
  - Geometric solids
  - Rectangular solids
  - Spheres
- Circles
  - Radius
  - Chords
  - Tangents
- Transformations
  - Reflections

Notes for use: One of the keys of geometry is learning deductive reasoning or how to do proofs. This can be a challenge to teach, so getting a teachers manual will really help here. The second main idea of geometry is getting used to thinking in space with shapes and the relationships between them. Lots of figures should be drawn.

1.7 Let us sum up

Evaluation is a an eminent part of any educational system. It determines the success and failure of the students, teachers and the entire curriculum. The unit above presented here a detail idea on evaluation and assessment, its importance, and scope. Further it has tried to explain the role of evaluation and also different techniques of evaluation. It has also tried to highlight different other methods currently in practice. Further it has tried to put up a discussion on different group of learners in the school may be gifted or retarded. The identification and their teaching strategies has been also presented in brief.

In the last part of this unit there is an analysis of textbooks in mathematics. Different guidelines and examples has been put to make the issue more clearer.

1.8 Check Your Progress

Now just try to go through the following questions and check your progress.

Q.1. How is measurement, assessment and evaluation are related. Present a brief discussion on it ?

Q.2. What is the need and importance of evaluation in the educational setup ?

Q.3 Describe different techniques in evaluation in mathematics?

Q.4 What is an Unit test in mathematics ? Prepare a blue-print and a question paper of 100 marks covering different topics of class IX of CBSE syllabus?

Q.5 Write in brief about the different observational and current alternative techniques of evaluation in mathematics education?

Q.6. Write the characteristics of gifted learners in mathematics. Describe the teaching strategy to meet their demands ?

Q.7 How will you identify retarded students in your class. Describe the strategies to help them overcome their learning deficiencies ?

Q.8. ‘ Analysis of mathematics textbook is most important else it will lead in creating misconception and confusion in the child’. Present the analysis mechanism of mathematics textbook of school?

1.9 Suggested Readings

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